

7. Planimetrija

7.1. Trougao

* Zbir unutrašnjih uglova: $\alpha + \beta + \gamma = 180^\circ$

* Zbir spoljašnjih uglova: $\alpha_1 + \beta_1 + \gamma_1 = 360^\circ$

* Zbir unutrašnjeg i odgovarajućeg spoljašnjeg ugla: $\alpha + \alpha_1 = 180^\circ$

* Srednja linija spaja sredine dveju strana

$$B_1C_1 = \frac{1}{2}BC, B_1C_1 \parallel BC$$

* Visina je normala iz temena na naspravnu stranicu. A' je podnožje visine h_a .

$$h_a \perp a, h_a = b \cdot \sin \alpha = c \cdot \sin \beta$$

* Težišna linija spaja teme i sredinu naspravne stranice

$$3. \quad t_a = AA_1, t_b = BB_1, t_c = CC_1, t_a \cap t_b \cap t_c = T$$

$$T - \text{težište trougla}, TA = \frac{2}{3}t_a, TA_1 = \frac{1}{3}t_a$$

* Centar O opisane kružnice je preseka simetrala stranica

$$O = s_a \cap s_b \cap s_c, OA = OB = OC = R - \text{poluprečnik opisane kružnice}$$

* Centar O_1 upisane kružnice je preseka simetrala uglova

$$O_1 = s_\alpha \cap s_\beta \cap s_\gamma$$

N, M, P - tačke u kojima upisana kružnica dodiruje stranice trougla

$$4. \quad O_1M \perp a, O_1N \perp b, O_1P \perp c$$

$$O_1M = O_1N = O_1P - \text{poluprečnik upisane kružnice}$$

$$AP = AN, BP = BM, CP = CN$$

* Obim trougla: $O = a + b + c$

* Površina trougla:

$$P = \frac{a \cdot h_a}{2} \quad \text{ili} \quad P = \frac{a \cdot b \cdot \sin \alpha}{2} = \frac{a \cdot c \cdot \sin \beta}{2}$$

$$P = \frac{a \cdot r}{2} + \frac{b \cdot r}{2} + \frac{c \cdot r}{2} = r \cdot \frac{a+b+c}{2} \Rightarrow P = \frac{r \cdot O}{2}$$

* Poluprečnik upisanog kruga: $r = \frac{2P}{O}$

* Sinusna teorema $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$

* Kosinusus teorema

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

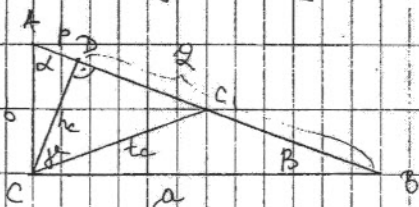
$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

1. $a : b : c = 4 : 5 : 8$ $a = 4x$ $4x + 5x + 8x = 68$ $a = 16$
 $O = 68 = a + b + c$ $b = 5x$ $17x = 68$ $b = 20$
 $c = 8x$ $x = 4$ $c = 32$

2. $a_1 = 5a$ $P = \frac{a \cdot h}{2}$ $P_1 = \frac{a_1 \cdot h_1}{2}$ $\frac{P}{P_1} = \frac{\frac{a \cdot h}{2}}{\frac{5a \cdot \frac{h}{2}}{4}} = \frac{2}{5}$
 $h_1 = \frac{h}{2}$
 $P : P_1 = ?$
 $P_1 = \frac{5a \cdot \frac{h}{2}}{2}$
 $P_1 = \frac{5a \cdot h}{4}$ $P : P_1 = 2 : 5$

* Pravnogi trougao



- centar opisanoj kruga C_1 , $CC_1 = t_c = AC_1 = BC_1 = R$

$$h_c^2 = p \cdot q, \quad b^2 = c \cdot p, \quad a^2 = c \cdot q$$

- Pitagorina teorema: $c^2 = a^2 + b^2$

$$\gamma = 90^\circ$$

$AB = c$ - hipotenuza

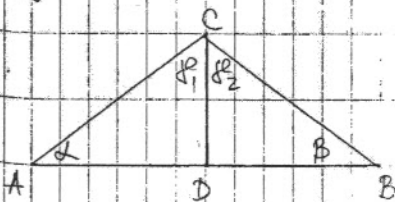
a, b - katete

$$\triangle ABC \sim \triangle ACD \sim \triangle CBD$$

$$p = AD, \quad q = BD$$

$$t_c = \frac{1}{2} c = \frac{1}{2} AB$$

* Jednokraki trougao

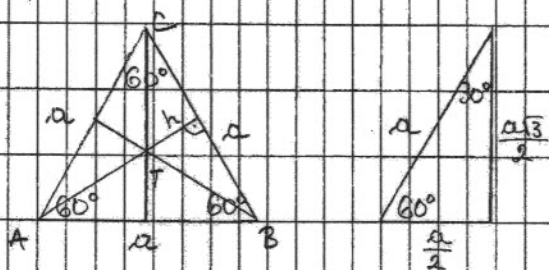


AB - osnovica

$AC = BC$ - kraci

$$\alpha = \beta, \quad \beta_1 = \beta_2 = \frac{\beta}{2}, \quad CD = t_c = h_c$$

* Jednakostraničan trougao



$$\alpha = \beta = \gamma = 60^\circ$$

$$h = t = \frac{a\sqrt{3}}{2}$$

$$P = \frac{a^2\sqrt{3}}{4}$$

3. $D = 72$

$$c = 30$$

a) $D = a + b + c$

$$72 = a + b + 30$$

$$a + b = 42$$

$$b = 42 - a$$

$$a = 18$$

$$b = 24$$

$$c^2 = a^2 + b^2$$

$$900 = a^2 + 1764 - 84a + a^2$$

$$2a^2 - 84a + 864 = 0 \quad | :2$$

$$a^2 - 42a + 432 = 0$$

$$a_{1,2} = \frac{42 \pm \sqrt{1764 - 1728}}{2} \rightarrow a_1 = 24 \quad b_1 = 18$$

$$\rightarrow a_2 = 18 \quad b_2 = 24$$

b) $P = \frac{a \cdot b}{2}$

$$P = \frac{18 \cdot 24}{2} = 216$$

d) $R = \frac{c}{2}$

$$R = 15$$

f) $b^2 = cp$

$$p = \frac{b^2}{c} = \frac{576}{30} = 19,2$$

c) $P = \frac{c \cdot h_c}{2} = \frac{a \cdot b}{2}$

$$\frac{30 \cdot h_c}{2} = 216$$

$$h_c = \frac{216}{15}$$

$$h_c = 14,4$$

e) $r = \frac{2P}{O}$

$$r = \frac{2 \cdot 216}{72 \cdot 30}$$

$$r = 6$$

$$a^2 = cq$$

$$q = \frac{a^2}{c} = \frac{324}{30} = 10,8$$

4. $c = 6$

$$a = 5$$

a) $a^2 = \left(\frac{c}{2}\right)^2 + h_c^2$

$$h_c^2 = 25 - 9$$

$$h_c^2 = 16$$

$$h_c = 4$$

$$p = \frac{c \cdot h_c}{2}$$

$$p = \frac{6 \cdot 4}{2}$$

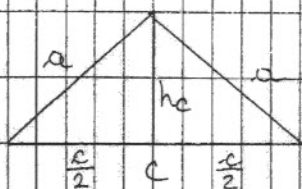
$$p = 12$$

$$p = \frac{a \cdot h_a}{2}$$

$$h_a = \frac{2p}{a}$$

$$h_a = \frac{24}{5}$$

$$h_a = 4,8$$



$$p = \frac{2bc}{5}$$

$$\frac{cb \sin \alpha}{2} = \frac{2bc}{5}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$p = \frac{c \cdot h_c}{2}$$

$$\sin \alpha = \frac{4}{5}$$

$$a^2 = b^2 + c^2 \pm 2bc \cdot \frac{3}{5}$$

$$h_c = b \cdot \sin \alpha$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$a = \sqrt{b^2 \pm \frac{6}{5}bc + c^2}$$

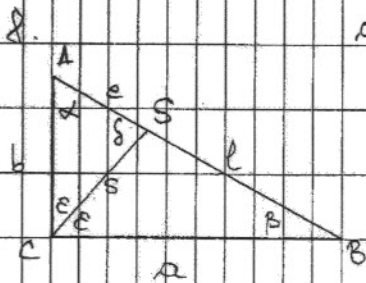
$$p = \frac{cb \sin \alpha}{2}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha}$$

$$\cos \alpha = \sqrt{1 - \frac{16}{25}}$$

$$\cos \alpha = \sqrt{\frac{9}{25}}$$

$$\cos \alpha = \pm \frac{3}{5}$$



$$a) \sin \alpha = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{s}{\sin \alpha} = \frac{b}{\sin \delta}$$

$$\cos \alpha = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\frac{s}{\sin \alpha} = \frac{b}{\sin (135^\circ - \alpha)}$$

$$\alpha = 45^\circ$$

$$s = \frac{b \sin \alpha}{\sin 135^\circ \cos \alpha - \cos 135^\circ \sin \alpha}$$

$$\delta = 180^\circ - \alpha - 45^\circ$$

$$s = \frac{b \cdot \sin \alpha}{\frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \sin \alpha}$$

$$\delta = 135^\circ - \alpha$$

$$s = \frac{b \sqrt{2} \cdot \frac{a}{\sqrt{a^2 + b^2}}}{\frac{b}{\sqrt{a^2 + b^2}} + \frac{a}{\sqrt{a^2 + b^2}}} = \frac{\frac{b \sqrt{2} \cdot a}{\sqrt{a^2 + b^2}}}{\frac{b + a}{\sqrt{a^2 + b^2}}} = \frac{ab \sqrt{2}}{a + b}$$

$$s = \frac{b \sin \alpha}{\frac{\sqrt{2}}{2} (\cos \alpha + \sin \alpha)}$$

$$s = \frac{2 b \sin \alpha}{\sqrt{2} (\cos \alpha + \sin \alpha)} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$s = \frac{\sqrt{2} b \sin \alpha}{\cos \alpha + \sin \alpha}$$

$$\begin{aligned} b) e^2 &= b^2 + s^2 - 2bs \cos \delta = \\ &= b^2 + \frac{a^2 b^2 \cdot 2}{(a+b)^2} - 2b \cdot \frac{ab \sqrt{2}}{a+b} \cos 45^\circ \\ &= \frac{b^2 (a^2 + 2ab + b^2) + 2a^2 b^2}{(a+b)^2} - \frac{2ab^2 \sqrt{2}}{a+b} \cdot \frac{\sqrt{2}}{2} = \\ &= \frac{a^2 b^2 + 2ab^3 + b^4 + 2a^2 b^2 - 2ab^2 (a+b)}{(a+b)^2} \end{aligned}$$

$$= \frac{3a^2b^2 + 2ab^3 + b^4 - 2a^2b^2 - 2ab^3}{(a+b)^2}$$

$$= \frac{a^2b^2 + b^4}{(a+b)^2} = \frac{b^2(a^2 + b^2)}{(a+b)^2}$$

$$e = \frac{\sqrt{b^2(a^2 + b^2)}}{(a+b)^2} = \frac{b\sqrt{a^2 + b^2}}{a+b}$$

$$BS = c - e = \sqrt{a^2 + b^2} - \frac{b\sqrt{a^2 + b^2}}{a+b}$$

$$BS = \sqrt{a^2 + b^2} \left(1 - \frac{b}{a+b}\right)$$

$$BS = \sqrt{a^2 + b^2} \cdot \frac{a}{a+b}$$

$$BS = \frac{a\sqrt{a^2 + b^2}}{a+b}$$

7.2. Krug

* Prečnik kruga $d = 2r$

* Obim kruga $O = 2r\pi$

* Površina kruga $P = r^2\pi$

* Perifernjski uglovi nad istom lukom su jednaki.

* Ugao između tangente i tetive jednak je perifernjskom uglu nad tom tetivom.

* Centralni ugao je dva puta veći od odgovarajućeg perifernjskog ugla

9. $a = 2\sqrt{3}$

a) $r = \frac{2}{3}h$

$$r = \frac{2}{3} \frac{a\sqrt{3}}{2}$$

$$r = \frac{a}{3}$$

$$r = 2$$

b) $b = 2r\pi$

$$b = 4\pi$$

$$P = r^2\pi$$

$$P = 4\pi$$

$$b : l = 360 : 30$$

$$4\pi : l = 12 : 1$$

$$12l = 4\pi$$

$$l = \frac{\pi}{3}$$

$$P : l = 360 : 30$$

$$4\pi : l = 12$$

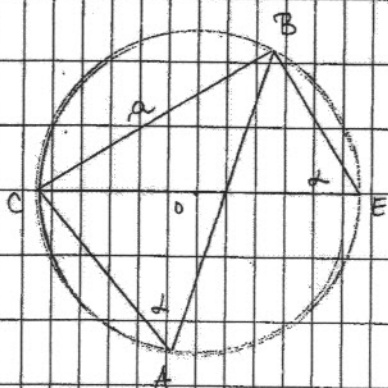
$$l = \frac{\pi}{3}$$

10. a) $\alpha = 90^\circ$

Dokazati da AB sadrži O.

Ugao α je periferijski ugao nad lukom AB. Odgovarajući centralni ugao je AOB (δ), dva puta veći od periferijskog $\delta = 2\alpha = 180^\circ$. Kraci ugla od 180° leže na pravoj pa prava koja prolazi kroz tačke A i B, tj. na kojoj leži stranica AB sadrži centar kruga O.

b) $\sin \alpha = \frac{a}{2R}$



$\triangle CBE$ je pravougli.

$\sphericalangle E$ je periferijski ugao nad lukom CB.

$\sphericalangle A$ je $\sphericalangle E$ - - - - -

$\sphericalangle A = \sphericalangle E = \alpha$

$\triangle EBC \rightarrow \sin \alpha = \frac{a}{CE}$

$CE = R$ - hipotenuza pravouglog trougla upisanog u kružnicu jednako je prečniku kruga.

11. $P = 6\sqrt{3}$

r

a) $r = h = \frac{a\sqrt{3}}{2}$

$r = \sqrt{3}$

$P_0 = r^2 \pi$

$P = \frac{3a^2\sqrt{3}}{2}$

$U_0 = 2r\pi$

$P_0 = 3\pi$

$U_0 = 2\sqrt{3}\pi$

$a^2 = \frac{2P}{3\sqrt{3}} = \frac{12\sqrt{3}}{3\sqrt{3}}$

$a^2 = 4$

$a = 2$

b) $h_c = r$

$c = 2r$

$P = \frac{c \cdot h_c}{2}$

$P = 3$

$h_c = \sqrt{3}$

$c = 2\sqrt{3}$

$P = \frac{2\sqrt{3} \cdot \sqrt{3}}{2}$

3. Četvorougao

* Zbir unutrašnjih uglova $\alpha + \beta + \gamma + \delta = 360^\circ$

* Spoljašnji ugao $\alpha_1 = 180^\circ - \alpha$

* Zbir spoljašnjih uglova $\alpha_1 + \beta_1 + \gamma_1 + \delta_1 = 360^\circ$

* Tetivni četvorougao je onaj oko koga se može upisati krug. Kod njega je $\alpha + \gamma = \beta + \delta = 180^\circ$

* Tangentni četvorougao je onaj u koji se može upisati krug. Kod njega je $AB + DC = AD + BC$

* Paralelogram

- Dijagonale se polove

- Naspramne stranice su paralelne i $AB = DC = a$, $AD = BC = b$.

- Naspramni uglovi su jednaki $\alpha = \gamma$, $\beta = \delta = 180^\circ - \alpha$

- Obim $O = 2a + 2b = 2(a + b)$

- Površina $P = a \cdot h_a$, $h_a = b \sin \alpha$

* Pravougaonik je paralelogram kod koga su svi uglovi pravi

- $\alpha = \beta = \gamma = \delta = 90^\circ$

- Površina $P = a \cdot b$

- Dužina dijagonale $d^2 = a^2 + b^2$

* Kvadrat je pravougaonik kod koga su sve četiri stranice jednake

$AB = BC = CD = DA = a$

- Obim $O = 4a$

- Površina $P = a^2$

* Romb je četvorougao kod koga su sve četiri stranice jednake

$AB = BC = CD = DA = a$

- Obim $O = 4a$

- Površina $P = \frac{d_1 \cdot d_2}{2}$

- Dijagonale d_1 i d_2 se seku pod pravim uglom

* Deltoid je četvorougao kod koga su dve susedne strane jednake

$AB = AD = a, CB = CD = b$

- Dijagonala d_1 polovi dijagonalu d_2 pod pravim uglom

* Trapez

- Srednja linija $m = \frac{a+b}{2}$

- Površina $P = \frac{a+b}{2} \cdot h$

- Kod jednakokrakog trapeza $l = \frac{a-b}{2} \quad h^2 = a^2 - l^2$

12. $P = 80$

$d_1 : d_2 = 5 : 4$

a) $4d_1 = 5d_2$

$d_1 = \frac{5d_2}{4}$

$P = \frac{d_1 d_2}{2}$

$P = \frac{5d_2^2}{8}$

$d_2^2 = \frac{8P}{5} = \frac{640}{5}$

$d_2^2 = 128$

$d_2 = \sqrt{128} = 8\sqrt{2}$

$d_1 = \frac{5 \cdot 8\sqrt{2}}{4} = 10\sqrt{2}$

$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$a^2 = 32 + 50$

$a^2 = 82$

$a = \sqrt{82}$

b) $P = a \cdot h$

$h = \frac{80}{\sqrt{82}} \cdot \frac{\sqrt{82}}{\sqrt{82}}$

$h = \frac{40\sqrt{82}}{41}$

$h = \frac{P}{a}$

$h = \frac{80\sqrt{82}}{82}$

13. $O = 14$

a) $O = 2a + 2b$

$P = a \cdot b$

$P = 12$

$14 = 2(a+b)$

$P = a \cdot (7-a)$

$a+b = 7$

$P = 7a - a^2$

$b = 7-a$

$-a^2 + 7a - 12 = 0$

$a_{1,2} = \frac{-7 \pm \sqrt{49 - 48}}{-2}$

$\rightarrow a_1 = 4 \quad b_1 = 3$

$\rightarrow a_2 = 3 \quad b_2 = 4$

$$a = 4$$
$$b = 3$$

$$b) r = \frac{d}{2}$$

$$d^2 = a^2 + b^2$$

$$d^2 = 25$$

$$d = 5$$

$$r = \frac{5}{2}$$

$$O = 2r\pi$$

$$O = 5\pi$$

$$P = r^2\pi$$

$$P = \frac{25}{4}\pi$$

axe

$$c) d = a\sqrt{2}$$

$$O = 4a$$

$$P = a^2$$

$$a = \frac{d}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$O = 10\sqrt{2}$$

$$P = 12,5$$

$$14. P = 16$$

$$x : y = 1 : 3$$

$$y = 3x$$

$$a) P = a^2$$

$$a = 4$$

$$a = x + y$$

$$4x = 4$$

$$x = 1$$

$$y = 3$$

$$b = y\sqrt{2}$$

$$b = 3\sqrt{2}$$

$$c = x\sqrt{2}$$

$$c = \sqrt{2}$$

$$P_{\square} = b \cdot a$$

$$P_{\square} = 6$$

$$b) d_2 = d = a\sqrt{2}$$

$$d = 4\sqrt{2}$$

$$d_2 = u + u$$

$$u = d_2 - u$$

$$u = \frac{b}{2} = \frac{3\sqrt{2}}{2}$$

$$u = 4\sqrt{2} - \frac{3\sqrt{2}}{2}$$

$$u = \frac{5\sqrt{2}}{2}$$

$$15. b : a = 3 : 4$$

$$4b = 3a$$

$$b = \frac{3a}{4}$$

$$P = 14$$

$$n = 2$$

$$P = \frac{a+b}{2} \cdot h$$

$$14 = \frac{a + \frac{3a}{4}}{2} \cdot 2$$

$$\frac{7a}{4} = 14$$

$$7a = 56$$

$$a = 8$$

$$b = 6$$

$$c^2 = h^2 + \left(\frac{a-b}{2}\right)^2$$

$$c^2 = 4 + \left(\frac{8-6}{2}\right)^2$$

$$c^2 = 4 + 1$$

$$c^2 = 5$$

$$c = \sqrt{5}$$

16. $a = 8$

$b = 4$

$c = 5$

a) $c^2 = h^2 + (a-b)^2$

$25 = h^2 + 16$

$h^2 = 9$

$h = 3$

$P = \frac{a+b}{2} \cdot h$

$P = \frac{12}{2} \cdot 3$

$P = 18$

b) $d_1^2 = h^2 + b^2$

$d_1^2 = 9 + 16$

$d_1^2 = 25$

$d_1 = 5$

$d_2^2 = a^2 + h^2$

$d_2^2 = 64 + 9$

$d_2^2 = 73$

$d_2 = \sqrt{73}$

17. $a = 6$

$c = 2$

$\alpha = 60^\circ$

$\beta = 135^\circ$

$x = \frac{c}{2}$

$x = 1$

$c^2 = h^2 + x^2$

$h^2 = 4 - 1$

$h^2 = 3$

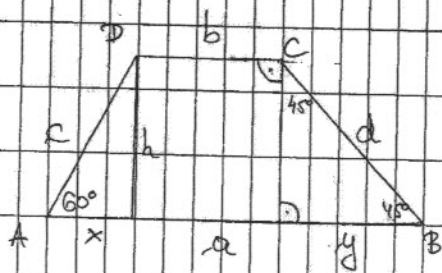
$h = \sqrt{3}$

$y = h = \sqrt{3}$

$b = a - (x + y)$

$b = 6 - 1 - \sqrt{3}$

$b = 5 - \sqrt{3}$

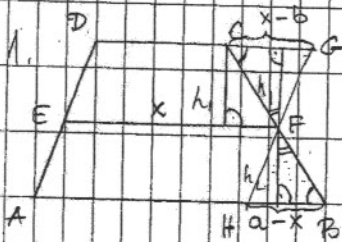


$P = \frac{a+b}{2} \cdot h$

$P = \frac{6 + 5 - \sqrt{3}}{2} \cdot \sqrt{3}$

$P = \frac{11\sqrt{3} - 3}{2}$

7.4. Zadaci za vežbu



$P_1 = P_2$

$\frac{x+b}{2} \cdot h_1 = \frac{a+x}{2} \cdot h_2$

$(x+b)h_1 = (a+x)h_2$

$\frac{h_1}{h_2} = \frac{a+x}{x+b}$

$\triangle HBF \sim \triangle GCF$

$\frac{h_1}{h_2} = \frac{CG}{HB} = \frac{x-b}{a-x}$

$\frac{a+x}{x+b} = \frac{x-b}{a-x}$

$a^2 - x^2 = x^2 - b^2$

$a^2 + b^2 = 2x^2$

$x^2 = \frac{a^2 + b^2}{2}$

$x = \sqrt{\frac{a^2 + b^2}{2}}$

$EF = \sqrt{\frac{a^2 + b^2}{2}}$

$$2. \quad a = 8$$

$$b = 4$$

$$d = 45^\circ$$

$$\beta = 30^\circ$$

$$x = h$$

$$y = \frac{d\sqrt{3}}{2}$$

$$d = 2h$$

$$y = \frac{2h\sqrt{3}}{2}$$

$$y = h\sqrt{3}$$

$$x + y + b = a$$

$$h + h\sqrt{3} + 4 = 8$$

$$h(1 + \sqrt{3}) = 4$$

$$h = \frac{4}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$h = \frac{4(1 - \sqrt{3})}{-2}$$

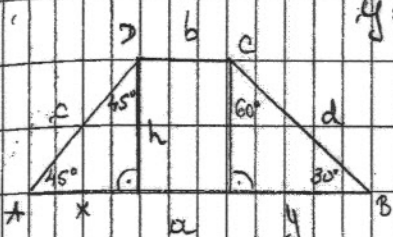
$$h = -2(1 - \sqrt{3})$$

$$h = 2(\sqrt{3} - 1)$$

$$P = \frac{a+b}{2} \cdot h$$

$$P = \frac{12}{2} \cdot 2(\sqrt{3} - 1)$$

$$P = 12(\sqrt{3} - 1)$$



$$3. \quad r = \frac{3}{2}$$

$$P = 15$$

$$h = 2r$$

$$h = 3$$

$$P = \frac{a+b}{2} \cdot h$$

$$a+b = \frac{2P}{h}$$

$$a+b = 10$$

$$a+b = 2a$$

$$c = 5$$

$$c^2 = h^2 + \left(\frac{a-b}{2}\right)^2$$

$$25 = 9 + \left(\frac{a-b}{2}\right)^2$$

$$\frac{a-b}{2} = 4$$

$$a-b = 8$$

$$a+b = 10$$

$$a-b = 8$$

$$2a = 18$$

$$a = 9$$

$$b = 1$$

$$d^2 = h^2 + \left(a - \frac{a-b}{2}\right)^2$$

$$d^2 = 9 + 25$$

$$d^2 = 34$$

$$d = \sqrt{34}$$

$$4. \quad u = 10$$

$$P_2 : P_1 = 3 : 5$$

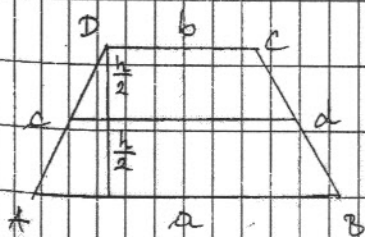
$$5P_2 = 3P_1$$

$$u = \frac{a+b}{2} = 10$$

$$a+b = 20$$

$$P_1 = \frac{a+u}{2} \cdot \frac{h}{2}$$

$$P_2 = \frac{b+u}{2} \cdot \frac{h}{2}$$



$$5. \quad \frac{b+u}{2} \cdot \frac{h}{2} = 3 \cdot \frac{a+u}{2} \cdot \frac{h}{2}$$

$$5b + 5u = 3a + 3u$$

$$5b - 3a = -2u$$

$$5b - 3a = -2 \cdot \frac{a+b}{2}$$

$$5b - 3a = -a - b$$

$$6b = 2a$$

$$a = \frac{6b}{2}$$

$$\frac{6b}{2} + b = 20$$

$$\frac{8b}{2} = 20$$

$$8b = 40$$

$$b = 5$$

$$a = 15$$

5. $R = 15$

$r = 6$

$R = \frac{c}{2}$

$c = 2R$

$c = 30$

$c^2 = a^2 + b^2$

$a^2 + b^2 = 900$

$r = \frac{a+b-c}{2}$

$6 = \frac{a+b-30}{2}$

$a+b-30 = 12$

$a+b = 42$

$b = 42 - a$

$a^2 + (42-a)^2 = 900$

$a^2 + 1764 - 84a + a^2 = 900$

$2a^2 - 84a + 864 = 0 \quad | :2$

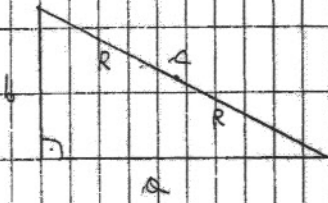
$a^2 - 42a + 432 = 0$

$a_{1,2} = \frac{42 \pm \sqrt{1764 - 1728}}{2}$

$a_1 = 24 \quad b_1 = 18$

$a_2 = 18 \quad b_2 = 24$

$a = 24, b = 18, c = 30$



6. $a = 30$

$r = 7,5$

$b^2 = \left(\frac{a}{2}\right)^2 + u^2$

$(2h-15)^2 = 225 + u^2$

$4h^2 - 60h + 225 - 225 - u^2 = 0$

$3h^2 - 60h = 0$

$u(3h-60) = 0$

$3h-60 = 0$

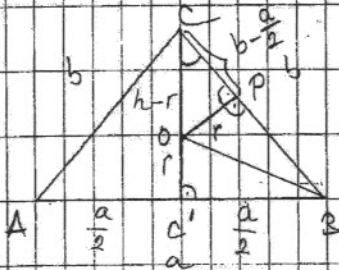
$3h = 60$

$h = 20$

$b = 25$

$p = \frac{a \cdot u}{2} = \frac{30 \cdot 20}{2}$

$p = 300$



$PBO \cong OCB$

$OPE \sim CO'B$

$b : \frac{a}{2} = (h-r) : r$

$b : 15 = (h-7,5) : 7,5$

$7,5b = 15h - 7,5 \cdot 15$

$b = \frac{15h - 7,5 \cdot 15}{7,5}$

$b = \frac{7,5 \cdot (2h-15)}{7,5}$

$b = 2h - 15$

7. $\alpha : \beta : \gamma = 2 : 3 : 7$

$\alpha = 2k$

$\beta = 3k$

$\gamma = 7k$

$\alpha + \beta + \gamma = 180^\circ$

$12k = 180^\circ$

$k = 15^\circ$

$\alpha = 30^\circ$

$\beta = 45^\circ$

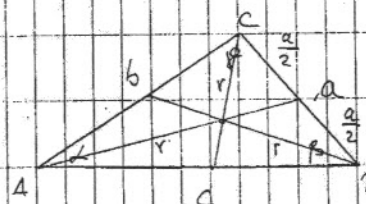
$\gamma = 105^\circ$

$\frac{a}{\sin \alpha} = 2R$

$2a = 2R$

$\frac{a}{\frac{1}{2}} = 2R$

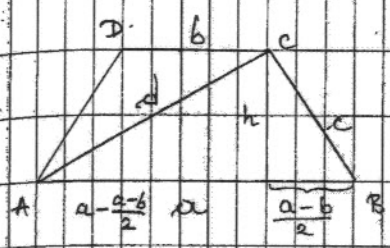
$R = a$



8. $P = 32$
 $h = 4$
 $a - b = 6$

$P = \frac{a+b}{2} \cdot h$
 $a+b = \frac{2P}{h}$
 $a+b = \frac{64}{4}$
 $a+b = 16$
 $\frac{a-b}{2} = 3$
 $2a = 22$
 $a = 11$
 $b = 5$

$d^2 = h^2 + \left(a - \frac{a-b}{2}\right)^2$
 $d^2 = 16 + (11-3)^2$
 $d^2 = 16 + 64$
 $d^2 = 80$
 $d = \sqrt{80}$
 $d = 4\sqrt{5}$



9. $D = 10\pi$
 $a:b = 3:4$
 $4a = 3b$
 $b = \frac{4a}{3}$

$D = 2r^2\pi$
 $r = 5$
 $r = \frac{d}{2}$
 $d = 10$

$d^2 = a^2 + b^2$
 $100 = a^2 + \frac{16a^2}{9}$
 $\frac{25a^2}{9} = 100 \sqrt{\quad}$
 $\frac{5a}{3} = 10$

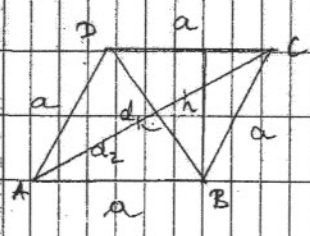
$a = 6$
 $b = 8$
 $P = a \cdot b$
 $P = 48$

10. $a = 5$
 $d_1 = 6$

$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$
 $\left(\frac{d_2}{2}\right)^2 = 25 - 9$
 $\left(\frac{d_2}{2}\right)^2 = 16$
 $\frac{d_2}{2} = 4$
 $d_2 = 8$

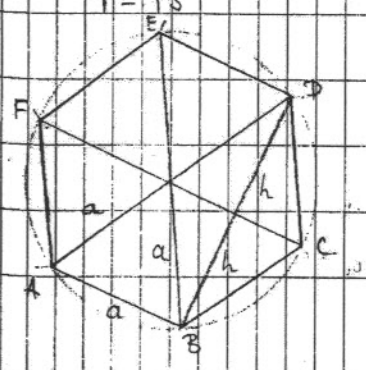
$P = \frac{d_1 \cdot d_2}{2}$
 $P = \frac{6 \cdot 8}{2} = 24$
 $P = a \cdot h$
 $h = \frac{P}{a} = \frac{24}{5}$

$r = \frac{h}{2}$
 $r = \frac{24}{10} = \frac{12}{5}$
 $P = r^2$
 $P = \frac{144\pi}{25}$



11. $O = 2\sqrt{3}\pi$
 $O = 2r^2\pi$
 $r = \sqrt{3}$

$r = h$
 $h = \sqrt{3}$
 $d = 2h$
 $d = 2\sqrt{3}$

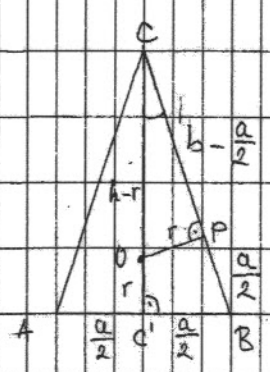


12. $a = 12$
 $r = 3$

$\triangle OPC \sim \triangle C'PB$
 $(h-r):r = b:\frac{a}{2}$
 $(h-3):3 = b:6$

$b^2 = h^2 + \left(\frac{a}{2}\right)^2$
 $(2h-6)^2 = h^2 + 36$
 $4h^2 - 24h + 36 = h^2 + 36$
 $3h^2 - 24h = 0$
 $h(3h-24) = 0$
 $3h-24 = 0$
 $3h = 24$
 $h = 8$
 $b = 10$

$O = a + 2b$
 $O = 32$
 $P = \frac{a \cdot h}{2}$
 $P = \frac{12 \cdot 8}{2}$
 $P = 48$

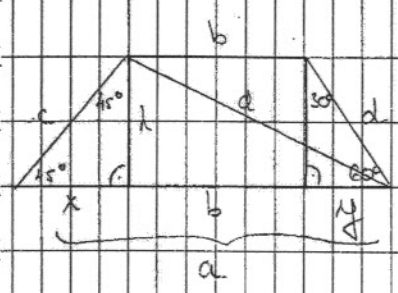


$6h - 18 = 3b$
 $b = \frac{6h-18}{3}$
 $b = \frac{3(2h-6)}{3}$
 $b = 2h-6$

13. $\alpha = 45^\circ$
 $\beta = 60^\circ$
 $h = \sqrt{3}$
 $O = 13 + \sqrt{6} - \sqrt{3}$

$x = h = \sqrt{3}$
 $h = \frac{d\sqrt{3}}{2}$
 $d = 2y$
 $h = \frac{2y\sqrt{3}}{2}$
 $y = 1$

$c = h\sqrt{2}$
 $a + b + c + d = 13 + \sqrt{6} - \sqrt{3}$
 $x + y + b + b + c + d = 13 + \sqrt{6} - \sqrt{3}$
 $\sqrt{3} + 1 + 2b + \sqrt{6} + 2 = 13 + \sqrt{6} - \sqrt{3}$
 $2b = 10 - 2\sqrt{3}$
 $b = \frac{2(5-\sqrt{3})}{2}$
 $b = 5 - \sqrt{3}$



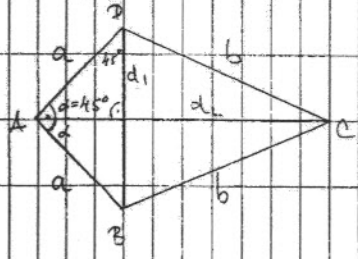
$d^2 = h^2 + (b+y)^2$
 $d^2 = 3 + (5-\sqrt{3}+1)^2$
 $d^2 = 3 + (6-\sqrt{3})^2$
 $d^2 = 3 + 36 - 12\sqrt{3} + 3$
 $d^2 = 42 - 12\sqrt{3}$
 $d = \sqrt{42 - 12\sqrt{3}}$

$a = x + y + b$
 $a = \sqrt{3} + 1 + 5 - \sqrt{3}$
 $a = 6$

14. $O = 6 + 2\sqrt{7}$
 $d_2 = 4\sqrt{2}$

$O = 2(a+b)$
 $a+b = \frac{2(3+\sqrt{7})}{2}$
 $a+b = 3 + \sqrt{7}$
 $b = 3 + \sqrt{7} - a$

$b^2 = a^2 + d_2^2 - 2ad_2 \cos \alpha$
 $(3 + \sqrt{7} - a)^2 = a^2 + d_2^2 - 2ad_2 \cdot \frac{\sqrt{2}}{2}$
 $(3 + \sqrt{7})^2 - 2(3 + \sqrt{7})a + a^2 = a^2 + d_2^2 - \sqrt{2}ad_2$
 $3 + 6\sqrt{7} + 7 - 6a - 2\sqrt{7}a = 32 - 8a$
 $-6 + 6\sqrt{7} + 2a - 2\sqrt{7}a = 0$



$$a(2-2\sqrt{7}) = 6-6\sqrt{7} \quad d_1 = a\sqrt{2} \quad P = \frac{d_1 d_2}{2} \quad P = 12$$

$$a(2-2\sqrt{7}) = 3(2-2\sqrt{7}) \quad d_1 = 3\sqrt{2} \quad P = \frac{3\sqrt{2} \cdot 4\sqrt{2}}{2}$$

$$a = 3$$

15

$$a_2 = \frac{8}{10} a_1$$

$$h = \frac{40\sqrt{2}}{\sqrt{41}}$$

$$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$

$$a^2 = \frac{d_1^2}{4} + \left(\frac{8d_1}{10}\right)^2$$

$$a^2 = \frac{d_1^2}{4} + \frac{16d_1^2}{100}$$

$$a^2 = \frac{41d_1^2}{100}$$

$$a = \frac{\sqrt{41}d_1}{10}$$

$$P = \frac{d_1 \cdot d_2}{2}$$

$$P = \frac{d_1 \cdot \frac{8}{10}d_1}{2}$$

$$P = \frac{4d_1^2}{10}$$

$$P = \frac{d_1 d_2}{2}$$

$$P = \frac{10\sqrt{2} \cdot 8\sqrt{2}}{2}$$

$$P = 80$$

$$P = a \cdot h$$

$$P = \frac{\sqrt{41}d_1}{10} \cdot \frac{40\sqrt{2}}{\sqrt{41}}$$

$$P = 4\sqrt{2}d_1$$

$$\frac{4d_1^2}{10} = 4\sqrt{2}d_1$$

$$d_1 = 10\sqrt{2}$$

$$d_2 = 8\sqrt{2}$$

6.

$$l = \frac{4\pi}{3}$$

$$\alpha = 60^\circ$$

$$B = 2L$$

$$B = 120^\circ$$

$$l = \frac{r\pi\alpha}{180}$$

$$r = \frac{180^\circ \cdot l}{\pi\alpha}$$

$$r = \frac{180^\circ \cdot \frac{4\pi}{3}}{120^\circ \cdot \pi}$$

$$r = 2$$

$$r = \frac{2}{3}h$$

$$h = 3$$

$$h = \frac{a\sqrt{3}}{2}$$

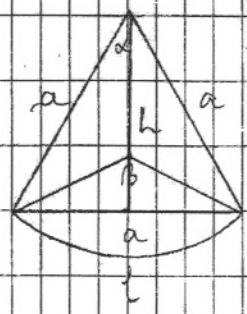
$$a = \frac{2h}{\sqrt{3}}$$

$$a = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$P = \frac{a^2\sqrt{3}}{4}$$

$$P = \frac{4 \cdot 3\sqrt{3}}{4}$$

$$P = 3\sqrt{3}$$



17.

$$a = 2\sqrt{2}$$

$$s = 3$$

$$\sin \alpha = \frac{a}{c} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos \alpha = \frac{b}{c} = \frac{b}{\sqrt{a^2+b^2}}$$

$$\frac{s}{\sin \alpha} = \frac{b}{\sin \delta}$$

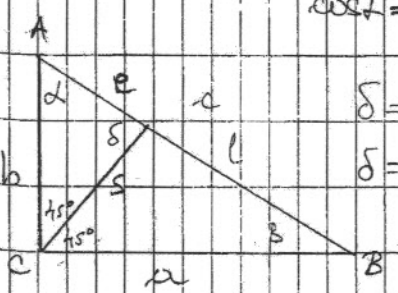
$$\frac{s}{\sin \alpha} = \frac{b}{\sin(135^\circ - \alpha)}$$

$$s = \frac{b \sin \alpha}{\sin 135^\circ \cos \alpha - \cos 135^\circ \sin \alpha}$$

$$s = \frac{b \sin \alpha}{\frac{\sqrt{2}}{2} \cos \alpha - \frac{\sqrt{2}}{2} \sin \alpha} = \frac{b\sqrt{2} \sin \alpha}{\cos \alpha + \sin \alpha}$$

$$\delta = 180^\circ - \alpha - 45^\circ$$

$$\delta = 135^\circ - \alpha$$



$$s = \frac{b\sqrt{2} \cdot \frac{a}{\sqrt{a^2+b^2}}}{\frac{b}{\sqrt{a^2+b^2}} + \frac{a}{\sqrt{a^2+b^2}}} = \frac{ab\sqrt{2}}{\frac{a+b}{\sqrt{a^2+b^2}}} = \frac{ab\sqrt{2}}{a+b}$$

$$P = \frac{a \cdot b}{2}$$

$$0 = \frac{2\sqrt{2} \cdot 6\sqrt{2}}{2}$$

$$3 = \frac{2\sqrt{2} \cdot b \cdot \sqrt{2}}{2\sqrt{2} + b}$$

$$P = 12$$

$$4b = 6\sqrt{2} - 3b$$

$$b = 6\sqrt{2}$$

18. $a = 5$
 $b = 3$
 $P = 6$

$$P = \frac{a \cdot ha}{2}$$

$$\sin \alpha = \frac{ha}{b}$$

$$c^2 = a^2 + b^2 - 2ab \cos \alpha$$

$$ha = \frac{2P}{a}$$

$$\sin \alpha = \frac{\frac{12}{5}}{3}$$

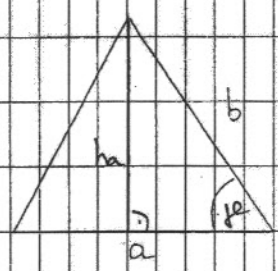
$$c^2 = 25 + 9 - 2 \cdot 5 \cdot 3 \cdot \frac{3}{5}$$

$$c^2 = 16$$

$$ha = \frac{12}{5}$$

$$\sin \alpha = \frac{4}{5}$$

~~$c = 4$~~ - ostroúhly trojúhelník



$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$\cos^2 \alpha = 1 - \frac{16}{25}$$

$$\cos^2 \alpha = \frac{9}{25}$$

$$\cos \alpha = \pm \frac{3}{5}$$

$$c^2 = 25 + 9 + 2 \cdot 5 \cdot 3 \cdot \frac{3}{5}$$

$$c^2 = 52$$

$$c = 2\sqrt{13}$$

19. $O = 36$
 $r = 3$

$$r = \frac{2P}{O}$$

$$P = 54$$

$$r = \frac{a+b-c}{2}$$

$$2P = 108$$

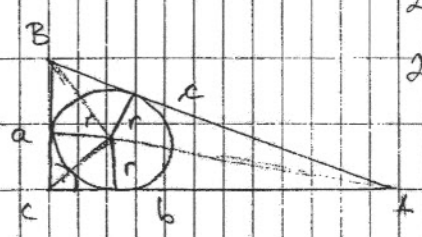
$$P = \frac{a \cdot b}{2}$$

$$a+b-c = 6$$

$$2P = 108$$

$$a \cdot b = 108$$

$$a+b+c = 36$$



$$2a+2b = 42$$

$$2b = 42 - 2a$$

$$b = \frac{42-2a}{2}$$

$$b = 21 - a$$

$$a(21-a) = 108$$

$$21a - a^2 - 108 = 0 \quad |(-1)$$

$$a^2 - 21a + 108 = 0$$

$$a_{1,2} = \frac{21 \pm \sqrt{441 - 432}}{2} \rightarrow a_1 = 12 \quad b_1 = 9$$

$$\rightarrow a_2 = 9 \quad b_2 = 12$$

$$a = 9, \quad b = 12$$

$$R = \frac{c}{2}$$

$$a + b + c = 36$$

$$R = 7,5$$

$$c = 36 - 9 - 12$$

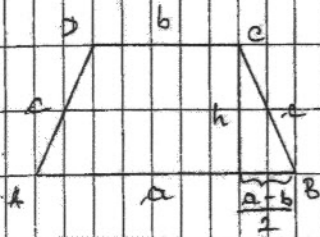
$$O = 2RT$$

$$c = 15$$

$$O = 15\pi$$

20. $c = b = 5$

$$h = 4$$



$$c^2 = h^2 + \left(\frac{a-b}{2}\right)^2$$

$$25 = 16 + \left(\frac{a-5}{2}\right)^2$$

$$\frac{a^2 - 10a + 25}{4} = 9$$

$$a^2 - 10a + 25 = 36$$

$$a^2 - 10a - 11 = 0$$

$$a_{1,2} = \frac{10 \pm \sqrt{100 + 44}}{2} \rightarrow a_1 = 11$$

$$\rightarrow a_2 = -1$$

$$P = \frac{a+b}{2} \cdot h$$

$$P = \frac{11+5}{2} \cdot 4$$

$$P = 32$$

21. $u = 5$

$$a + b = 20$$

$$O = a + b + 2c$$

$$\frac{a+b}{2} = 5$$

$$c = 5$$

$$O = 20$$

$$a + b = 10$$

22. $\beta = \frac{\pi}{3} = 60^\circ$

$$O = 20$$

$$c = d = \frac{a}{2}$$

$$d = a - b$$

$$\frac{a}{2} = a - b$$

$$a = 2a - 2b$$

$$a = 2b$$

$$b = \frac{a}{2}$$

$$b = c = d = \frac{a}{2}$$

$$O = a + b + c + d$$

$$a + \frac{3a}{2} = 20$$

$$\frac{5a}{2} = 20$$

$$5a = 40$$

$$a = 8$$

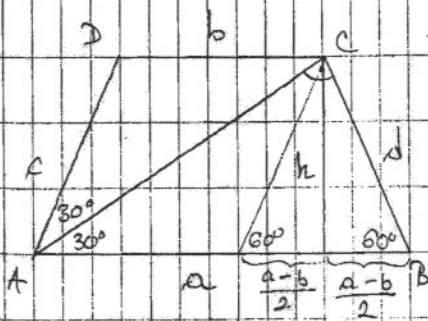
$$b = 4$$

$$h = \frac{a\sqrt{3}}{2} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$$

$$P = \frac{a+b}{2} \cdot h$$

$$P = \frac{12}{2} \cdot 2\sqrt{3}$$

$$P = 12\sqrt{3}$$



23. $n=5$

$d_1 + d_2 = 14$

$d_2 = 14 - d_1$

$d^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$25 = \frac{d_1^2}{4} + \frac{196 - 28d_1 + d_1^2}{4}$

$2d_1^2 - 28d_1 + 196 = 100$

$2d_1^2 - 28d_1 + 96 = 0 \quad | :2$

$d_1^2 - 14d_1 + 48 = 0$

$d_{1,2} = \frac{14 \pm \sqrt{196 - 192}}{2} \rightarrow d_1 = 8 \quad d_2 = 6$

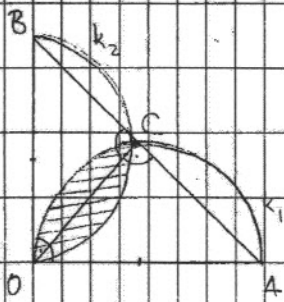
$\rightarrow d_1 = 6 \quad d_2 = 8$

$P = \frac{d_1 d_2}{2}$

$P = \frac{8 \cdot 6}{2}$

$P = 24$

24.



a) $\sphericalangle OCA = \sphericalangle OCB = 90^\circ \Rightarrow$

\Rightarrow tačke A, B i C su kolinearne

b) $OA = R = 4 \quad r = 2$

$P = \frac{r^2 \pi}{2} - P_{\Delta OCA}$

$P_{\Delta OCA} = 2r \cdot \frac{r}{2}$

$P = \frac{4\pi}{2} - 4$

$P_{\Delta OCA} = r^2 = 4$

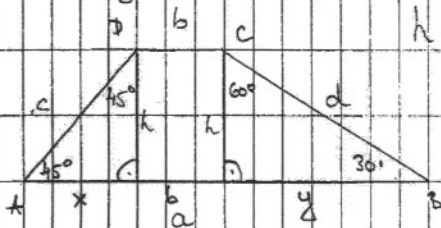
$P = 2\pi - 4$

26. $AB = 8$

$CD = 4$

$\alpha = \frac{\pi}{4} = 45^\circ$

$\beta = \frac{\pi}{6} = 30^\circ$



$x = h$

$y = \frac{2h\sqrt{3}}{2}$

$y = h\sqrt{3}$

$x + y + b = a$

$h + h\sqrt{3} + 4 = 8$

$h(1 + \sqrt{3}) = 4$

$h = \frac{4}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{4 - 4\sqrt{3}}{1 - 3} = \frac{2(2 - 2\sqrt{3})}{-2}$

$h = 2\sqrt{3} - 2$

$h = 2(\sqrt{3} - 1)$

$P = \frac{a+b}{2} \cdot h$

$P = \frac{12}{2} \cdot 2(\sqrt{3} - 1)$

$P = 12(\sqrt{3} - 1)$