40. $2^x \cdot 3^{y-2} = 4$

$2^x + \sqrt{3^{2y}} = 13$

$2^x \cdot 3^y \cdot 3^{-2} = 4$

$2^x \cdot 3^y = \frac{4}{\frac{1}{9}} = 36$ $\qquad 2^x = a$

$2^x + 3^y = 13$ $\qquad 3^y = b$

$a \cdot b = 36$

$a + b = 13$

$a = 13 - b$

$(13 - b) \cdot b = 36$

$13b - b^2 - 36 = 0$

$b_{1,2} = \frac{-13 \pm \sqrt{169 - 144}}{-2} \rightarrow b_1 = 4 \quad a_1 = 9$; $\rightarrow b_2 = 9 \quad a_2 = 4$

$3^{y_1} = 4$

$\boxed{y_1 = \log_3 4}$

$2^{x_1} = 9$

$\boxed{x_1 = \log_2 9}$

$3^{y_2} = 9$

$\boxed{y_2 = 2}$

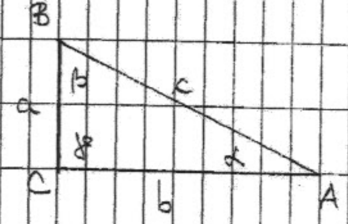
$2^{x_2} = 4$

$\boxed{x_2 = 2}$

4. Trigonometrija

4.1. Osnovne trigonometrijske funkcije

$180° = \pi$



$\sin\alpha = \frac{a}{c} \quad \cos\alpha = \frac{b}{c} \quad tg\alpha = \frac{a}{b} \quad ctg\alpha = \frac{b}{a}$

$\sin\beta = \frac{b}{c} \quad \cos\beta = \frac{a}{c} \quad tg\beta = \frac{b}{a} \quad ctg\beta = \frac{a}{b}$

$\sin\alpha = \cos\beta \quad \sin\beta = \cos\alpha \quad tg\alpha = ctg\beta \quad tg\beta = ctg\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \qquad \left(\frac{a+b}{c}\right)^2 = \frac{(a+b)^2}{c^2} = \frac{c^2}{c^2} = 1$

$-1 \leq \sin\alpha \leq 1 \qquad -1 \leq \cos\beta \leq 1 \qquad tg(\pi - \alpha) = -tg\alpha \qquad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$ 3

$\sin(\pi - \alpha) = \sin\alpha \qquad \sin(\pi + \alpha) = -\sin\alpha \qquad tg(-\alpha) = -tg\alpha \qquad tg\left(\frac{\pi}{2} - \alpha\right) = ctg\alpha$

$\sin(-\alpha) = -\sin\alpha \qquad \sin(2\pi - \alpha) = -\sin\alpha \qquad ctg(\pi - \alpha) = -ctg\alpha \qquad ctg\left(\frac{\pi}{2} - \alpha\right) = tg\alpha$

$\cos(\pi - \alpha) = -\cos\alpha \qquad \cos(\pi + \alpha) = -\cos\alpha \qquad ctg(-\alpha) = -ctg\alpha$

$\cos(-\alpha) = \cos\alpha \qquad \cos(2\pi - \alpha) = \cos\alpha \qquad \sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

α	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$tg\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	/	0
$ctg\alpha$	/	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	/

	$0°<\alpha<90°$	$90°<\alpha<180°$	$180°<\alpha<270°$	$270°<\alpha<360°$
$\sin\alpha$	+	+	−	−
$\cos\alpha$	+	−	−	+

$\sin(\alpha+2k\pi)=\sin\alpha$, $\cos(\alpha+2k\pi)=\cos\alpha$

$tg(\alpha+k\pi)=tg\alpha$, $ctg(\alpha+k\pi)=ctg\alpha$, $k\in Z$

1. $\cos 945° = ?$

$\cos 945° = \cos(720°+225°)$

$\cos 225° = \cos(180°+45°) = -\cos 45°$

$-\cos 45° = -\frac{\sqrt{2}}{2}$

2. $tg\alpha = -\sqrt{3}$

$\alpha = ?$

$\sqrt{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = tg\frac{\pi}{3}$

$\alpha = \frac{2\pi}{3} + k\pi, \ k\in Z$

3. $\sin\alpha = ?$ $\quad \cos\alpha = -\frac{3}{5}, \ \pi < \alpha < \frac{3\pi}{2}$ $\quad tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{4}{3}$

$tg\alpha = ?$ $\quad \sin^2\alpha = 1-\cos^2\alpha$

$ctg\alpha = ?$ $\quad \sin^2\alpha = \frac{25}{25}-\frac{9}{25}$ $\quad ctg\alpha = \frac{1}{tg\alpha} = \frac{3}{4}$

$\sin^2\alpha = \frac{16}{25}$

$\sin\alpha = -\frac{4}{5}$

4. $tg\alpha = -\frac{5}{12}$, $\frac{\pi}{2} < \alpha < \pi$

$\sin\left(\frac{9\pi}{2} - \alpha\right) = ?$

$\sin\left(\frac{9\pi}{2} - \alpha\right) = \sin\left(4\pi + \frac{\pi}{2} - \alpha\right) = \sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

$\frac{\sin\alpha}{\cos\alpha} = -\frac{5}{12}$

$\frac{\sin^2\alpha}{\cos^2\alpha} = \frac{25}{144}$

$144(1-\cos^2\alpha) = 25\cos^2\alpha$

$144 - 144\cos^2\alpha - 25\cos^2\alpha = 0$

$169\cos^2\alpha = 144$

$\cos^2\alpha = \frac{144}{169}$

$\cos\alpha = -\frac{12}{13}$

5. $ctg\alpha = 0{,}75$, $\pi < \alpha < \frac{3\pi}{2}$

$\sin\alpha = ?$

$\cos\alpha = ?$

$tg\alpha = ?$

$tg\left(\frac{15\pi}{2} - \alpha\right) = ?$

$tg\left(\frac{15\pi}{2} - \alpha\right) = tg\left(7\pi + \frac{\pi}{2} - \alpha\right) =$

$= tg\left(\frac{\pi}{2} - \alpha\right) = ctg\alpha = \frac{3}{4}$

$ctg\alpha = \frac{3}{4}$

$tg\alpha = \frac{1}{ctg\alpha}$

$tg\alpha = \frac{4}{3}$

$tg\alpha = \frac{\sin\alpha}{\cos\alpha}$

$\frac{\sin^2\alpha}{\cos^2\alpha} = \frac{16}{9}$

$9(1-\cos^2\alpha) = 16\cos^2\alpha$

$9 = 25\cos^2\alpha$

$\cos^2\alpha = \frac{9}{25}$

$\cos\alpha = -\frac{3}{5}$

$\sin^2\alpha = 1 - \cos^2\alpha$

$\sin^2\alpha = \frac{25}{25} - \frac{9}{25}$

$\sin^2\alpha = \frac{16}{25}$

$\sin = -\frac{4}{5}$

6. $\sin\frac{3\pi}{2} + \cos\frac{5\pi}{6} - tg\frac{3\pi}{4} + ctg\frac{7\pi}{6} =$

$= \cancel{-1} - \cos\left(\frac{4}{6}\pi + \frac{\pi}{6}\right) - \cancel{(-1)} + ctg\left(\pi + \frac{\pi}{6}\right) =$

$= -\frac{\sqrt{3}}{2} + \sqrt{3} = \frac{\sqrt{3}}{2}$

7. $2\cos 120° - 3tg\,150° + 2\sin 210° - ctg\,135° =$

$= 2\cos(180° - 60°) - 3tg(180° - 30°) + 2\sin(180° + 30°) - ctg(180° - 45°) =$

$= -2\cos 60° + 3tg\,30° - 2\sin 30° + ctg\,45° = -1 + \sqrt{3} - 1 + 1 = \sqrt{3} - 1$

8. $\cos\alpha = -\frac{1}{2}$

$\alpha = \frac{\pi}{2} + \frac{\pi}{6} + 2k\pi = \frac{2\pi}{3} + 2k\pi, \ k \in \mathbb{Z}$

$\alpha = \pi + \frac{\pi}{3} + 2k\pi = \frac{4\pi}{3} + 2k\pi, \ k \in \mathbb{Z}$

9. $\operatorname{tg} x = \sqrt{3}$

$x = \frac{\pi}{3} + k\pi, \ k \in \mathbb{Z}$

10. $\sin\alpha = -\frac{\sqrt{2}}{2}$

$\alpha = \pi + \frac{\pi}{4} + 2k\pi = \frac{5\pi}{4} + 2k\pi$

$\alpha = \pi + \frac{\pi}{2} + \frac{\pi}{4} + 2k\pi = \frac{7\pi}{4} + 2k\pi$

11. $\sin x = \sin 13^\circ$

$x = 13^\circ + 360^\circ k, \ k \in \mathbb{Z}$

$x = 180^\circ - 13^\circ + 360^\circ k = 167^\circ + 360^\circ k, \ k \in \mathbb{Z}$

12. $2\cos^2 x - 7\cos x = 4 \qquad \cos x = t$

$2t^2 - 7t - 4 = 0$

$t_{1,2} = \frac{7 \pm \sqrt{49+32}}{4} \to \cancel{t_1 = 4}, \quad t_2 = -\frac{1}{2}$

$\cos x = -\frac{1}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi}{6} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} + 2k\pi, \ k \in \mathbb{Z}$

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} + 2k\pi, \ k \in \mathbb{Z}$

$x \in \left\{\frac{2\pi}{3} + 2k\pi \mid k \in \mathbb{Z}\right\} \cup \left\{\frac{4\pi}{3} + 2k\pi \mid k \in \mathbb{Z}\right\}$

13. $\sin 5x - \sqrt{3}\cos 5x = \sqrt{3}$

$\sin 5x - \sqrt{3} = \sqrt{3}\sqrt{1-\sin^2 5x} \quad /^2$

$\sin^2 5x - 2\sqrt{3}\sin 5x + 3 = 3 - 3\sin^2 5x$

$4\sin^2 5x - 2\sqrt{3}\sin 5x = 0$

$2\sin 5x\,(2\sin 5x - \sqrt{3}) = 0$

$2\sin 5x = 0 \quad \vee \quad 2\sin 5x - \sqrt{3} = 0$

$\sin 5x = 0 \qquad 2\sin 5x = \sqrt{3}$

$5x = 2k\pi \qquad \sin 5x = \frac{\sqrt{3}}{2}$

$x = \frac{2k\pi}{5}$

$5x = \frac{\pi}{3} + 2k\pi \qquad 5x = \frac{2\pi}{3} + 2k\pi$

$x = \frac{\frac{\pi}{3} + 2k\pi}{5} \qquad x = \frac{2\pi}{15} + \frac{2k\pi}{5}$

14. $1 + \sin 2x = (\sin 2x - \cos 2x)^2$

$1 + \sin 2x = \sin^2 2x - 2\sin 2x\cos 2x + \cos^2 2x$

$1 + \sin 2x = 1 - 2\sin 2x\cos 2x$

$\sin 2x + 2\sin 2x\cos 2x = 0$

$\sin 2x\,(1 + 2\cos 2x) = 0$

$\sin 2x = 0 \quad \vee \quad 1 + 2\cos 2x = 0$

$2x = k\pi \qquad 2\cos 2x = -1$

$x = \frac{k\pi}{2} \qquad \cos 2x = -\frac{1}{2}$

$2x = \frac{2\pi}{3} + 2k\pi \qquad 2x = \frac{4\pi}{3} + 2k\pi$

$x = \frac{\pi}{3} + k\pi \qquad x = \frac{2\pi}{3} + k\pi$

$x \in \left\{\frac{k\pi}{2} \mid k \in \mathbb{Z}\right\} \cup \left\{\frac{\pi}{3} + k\pi \mid k \in \mathbb{Z}\right\} \cup \left\{\frac{2\pi}{3} + k\pi \mid k \in \mathbb{Z}\right\}$

15. $\sin^2 x + 2\sin x\cos x - \cos^2 x = 1$

$1 - \cos^2 x + 2\sin x\cos x - \cos^2 x = 1$

$2\sin x\cos x - 2\cos^2 x = 0$

$2\cos x\,(\sin x - \cos x) = 0$

$2\cos x = 0 \quad \vee \quad \sin x - \cos x = 0$

$\cos x = 0 \qquad \sin x = \cos x$

$x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$ $\sin x = \sin\left(\frac{\pi}{2} - x\right)$

$$x = \frac{\pi}{2} - x + 2k\pi$$

$$2x = \frac{\pi}{2} + 2k\pi$$

$$x = \frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}$$

10. $\operatorname{tg} x \left(\operatorname{tg} x + \frac{1}{\cos x}\right) = 1$

$$\frac{\sin x}{\cos x}\left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right) = 1 \qquad \cos x \neq 0$$

$$\frac{\sin x}{\cos x}\left(\frac{\sin x + 1}{\cos x}\right) = 1$$

$$\frac{\sin x(\sin x + 1)}{\cos^2 x} = 1$$

$$\sin^2 x + \sin x = \cos^2 x$$

$$\sin^2 x + \sin x = 1 - \sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0 \qquad \sin x = t$$

$$2t^2 + t - 1 = 0$$

$$t_{1,2} = \frac{-1 \pm \sqrt{1+8}}{4} \Rightarrow t_1 = \frac{1}{2},\ t_2 = -1$$

$$\sin x = -1 \quad \vee \quad \sin x = \frac{1}{2}$$

$$\cancel{x = \frac{3\pi}{2} + 2k\pi} \qquad x = \frac{\pi}{6} + 2k\pi$$

$$x = \frac{5\pi}{6} + 2k\pi$$

$$x \in \left\{\frac{\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\right\} \cup \left\{\frac{5\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\right\}$$

4.2. Trigonometrijski identiteti

Adicione formule

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha$

$$tg\,2\alpha=\frac{\sin 2\alpha}{\cos 2\alpha}=\frac{2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{\frac{2\sin\alpha\cos\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha}}=\frac{2\,tg\,\alpha}{1-tg^2\alpha}$$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$1+\cos\alpha=2\cos^2\frac{\alpha}{2}$

$1-\cos\alpha=2\sin^2\frac{\alpha}{2}$

17. $\alpha+\beta=60^\circ$

$\cos\alpha=\frac{11}{13}$

$\cos\beta=?$

$\beta=60^\circ-\alpha$

$\cos\beta=\cos(60^\circ-\alpha)$

$\cos\beta=\cos 60^\circ\cos\alpha+\sin 60^\circ\sin\alpha$

$\sin\alpha=\pm\sqrt{1-\cos^2\alpha}$

$$\sin\alpha=\pm\sqrt{1-\frac{121}{169}}=\pm\sqrt{\frac{169-121}{169}}$$

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$\sin\alpha=\pm\frac{4\sqrt{3}}{13}$

$$\cos\beta=\frac{1}{2}\cdot\frac{11}{13}\pm\frac{\sqrt{3}}{2}\cdot\frac{4\sqrt{3}}{13}$$

$$\cos\beta=\frac{11}{26}\pm\frac{12}{26}$$

$$\cos\beta=\frac{23}{26}\ \vee\ \cos\beta=-\frac{1}{26}$$

$$\cos\beta\in\left\{\frac{23}{26},-\frac{1}{26}\right\}$$

18. $\alpha+\beta=135°$ $\quad \sin(135°-\beta)=\frac{15}{17}$

$\sin\alpha=\frac{15}{17}$ $\quad \sin 135° \cos\beta - \cos 135° \sin\beta = \frac{15}{17}$

$\sin\beta+\cos\beta=?$ $\quad \frac{\sqrt{2}}{2}\cos\beta + \frac{\sqrt{2}}{2}\sin\beta = \frac{15}{17}$

$\cos\beta+\sin\beta = \frac{\frac{15}{17}}{\frac{\sqrt{2}}{2}} = \frac{30}{17\sqrt{2}} = \frac{15\sqrt{2}}{17}$

19. $\sin(x+30°)+\cos(x+60°)=1+\cos 2x$

$\sin x\cos 30°+\cos x\sin 30°+\cos x\cos 60°-\sin x\sin 60°=2\cos^2 x$

$\cancel{\frac{\sqrt{3}}{2}\sin x}+\frac{1}{2}\cos x+\frac{1}{2}\cos x-\cancel{\frac{\sqrt{3}}{2}\sin x}=2\cos^2 x$

$\cos x=2\cos^2 x$

$2\cos^2 x-\cos x=0$

$\cos x(2\cos x-1)=0$

$\cos x=0 \quad \vee \quad 2\cos x-1=0$

$2\cos x=1$

$x=\frac{\pi}{2}+2k\pi$ $\quad \cos x=\frac{1}{2}$

$x=\frac{3\pi}{2}+2k\pi$

$x=\frac{\pi}{3}+2k\pi$

$x=\frac{5\pi}{3}+2k\pi \quad k\in\mathbb{Z}$

20. $\sin 3x=4\sin x\cos 2x$

$\sin(x+2x)=4\sin x\cos 2x$

$\sin x\cos 2x+\cos x\sin 2x=4\sin x\cos 2x$

$3\sin x\cos 2x=\cos x\sin 2x$

$2\sin x\cos^2 x=3\sin x\cos 2x$

$2\sin x\cos^2 x=3\sin x(2\cos^2 x-1)$

$2\sin x\cos^2 x-6\sin x\cos^2 x+3\sin x=0$

$-4\sin x\cos^2 x+3\sin x=0$

$\sin x(-4\cos^2 x+3)=0$

$\sin x=0 \quad \vee \quad -4\cos^2 x+3=0$

$x=k\pi$ $\quad 4\cos^2 x=3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \pm\frac{\pi}{6} + 2k\pi$

$x = \pm\frac{5\pi}{6} + 2k\pi, \ k \in \mathbb{Z}$

21. $\frac{1 - 2\sin^2\alpha}{1 + \sin 2\alpha} = \frac{1 - \operatorname{tg}\alpha}{1 + \operatorname{tg}\alpha} \qquad \alpha \neq \frac{3\pi}{4} + k\pi \wedge \alpha \neq \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}$

$$\frac{1 - 2\sin^2\alpha}{1 + \sin 2\alpha} = \frac{\cos^2\alpha + \sin^2\alpha - 2\sin^2\alpha}{\cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha} = \frac{\cos^2\alpha - \sin^2\alpha}{(\cos\alpha + \sin\alpha)^2} =$$

$$= \frac{(\cos\alpha - \sin\alpha)\cancel{(\cos\alpha + \sin\alpha)}}{(\cos\alpha + \sin\alpha)^{\cancel{2}}} = \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} = \frac{\cancel{\cos\alpha}\left(1 - \frac{\sin\alpha}{\cos\alpha}\right)}{\cancel{\cos\alpha}\left(1 + \frac{\sin\alpha}{\cos\alpha}\right)} =$$

$$= \frac{1 - \operatorname{tg}\alpha}{1 + \operatorname{tg}\alpha}$$

22. $\frac{\sin^4\alpha + 2\sin\alpha\cos\alpha - \cos^4\alpha}{\operatorname{tg}2\alpha - 1} = \cos 2\alpha \qquad \alpha \neq \frac{\pi}{8} + \frac{k\pi}{2} \wedge \alpha \neq \frac{\pi}{4} + \frac{k\pi}{2}, \ k \in \mathbb{Z}$

$$\frac{(\sin^4\alpha - \cos^4\alpha) + \sin 2\alpha}{\frac{\sin 2\alpha}{\cos 2\alpha} - 1} = \frac{(\sin^2\alpha - \cos^2\alpha)(\sin^2\alpha + \cos^2\alpha) + \sin 2\alpha}{\frac{\sin 2\alpha - \cos 2\alpha}{\cos 2\alpha}} =$$

$$= \frac{-\cos 2\alpha + \sin 2\alpha}{\frac{\sin 2\alpha - \cos 2\alpha}{\cos 2\alpha}} = \frac{\cos 2\alpha\,\cancel{(\sin 2\alpha - \cos 2\alpha)}}{\cancel{\sin 2\alpha - \cos 2\alpha}} = \cos 2\alpha$$

23. $\operatorname{ctg}2x - \operatorname{tg}2x = \frac{2}{3}\operatorname{tg}4x$

$\frac{1}{\operatorname{tg}2x} - \operatorname{tg}2x = \frac{2}{3}\operatorname{tg}4x$

$\frac{1}{\operatorname{tg}2x} - \operatorname{tg}2x = \frac{2}{3} \cdot \frac{2\operatorname{tg}2x}{1 - \operatorname{tg}^2 2x} \qquad \operatorname{tg}2x = t$

$\frac{1}{t} - t = \frac{4}{3} \cdot \frac{t}{1 - t^2} \quad \Big| \cdot t$

$1-t^2=\frac{4}{3}\frac{t^2}{1-t^2} \quad |\cdot(1-t^2)$

$(1-t^2)^2=\frac{4}{3}t^2 \quad |\cdot 3$

$3(1-t^2)=4t^2$

$3(1-2t^2+t^4)=4t^2$

$3-6t^2+3t^4-4t^2=0$

$3t^4-10t^2+3=0 \qquad t^2=z$

$3z^2-10z+3=0$

$z_{1,2}=\frac{10\pm\sqrt{100-36}}{6}$ $\nearrow z_1=3$ $\searrow z_2=\frac{1}{3}$

$t_{1,2}=\pm\sqrt{3} \qquad t_{3,4}=\pm\frac{1}{\sqrt{3}}$

$tg\,2x=\pm\sqrt{3} \qquad tg\,2x=\pm\frac{1}{\sqrt{3}}$

$2x=\pm\frac{\pi}{3}+k\pi \qquad 2x=\pm\frac{\pi}{6}+k\pi$

$x=\pm\frac{\pi}{6}+\frac{k\pi}{2} \qquad x=\pm\frac{\pi}{12}+\frac{k\pi}{2}, \quad k\in\mathbb{Z}$

* Transformacije zbira i razlike

$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

24. a) $1+\sin 2x=(\sin 5x+\cos 5x)^2$

$1+\sin 2x=\sin^2 5x+2\sin 5x\cos 5x+\cos^2 5x$

$1+\sin 2x=1+\sin 10x$

$\sin 10x-\sin 2x=0$

$2\sin\frac{8x}{2}\cos\frac{12x}{2}=0$

$2\sin 4x\cos 6x=0$

$\sin 4x\cos 6x=0$

$\sin 4x=0 \lor \cos 6x=0$

$4x=2k\pi$ $\qquad$ $6x=\frac{\pi}{2}+2k\pi$

$x=\frac{k\pi}{2}$ $\qquad$ $x=\frac{\pi}{12}+\frac{k\pi}{6}$

b) $\cos\left(\frac{\pi}{2}+5x\right)+\sin x=2\cos 3x$

$0\cdot\cos 5x-1\cdot\sin 5x+\sin x=2\cos 3x$

$-\sin 5x+\sin x=2\cos 3x$

$-2\sin 2x\cos 3x=2\cos 3x$

$-\sin 2x\cos 3x=\cos 3x$

$\cos 3x+\sin 2x\cos 3x=0$

$\cos 3x(1+\sin 2x)=0$

$\cos 3x=0 \lor 1+\sin 2x=0$

$3x=\frac{\pi}{2}+k\pi$ $\qquad$ $\sin 2x=-1$

$x=\frac{\pi}{6}+\frac{k\pi}{3}$ $\qquad$ $2x=\frac{3\pi}{2}+2k\pi$

$x=\frac{3\pi}{4}+k\pi,\ k\in\mathbb{Z}$

c) $\cos\left(\frac{\pi}{2}-3x\right)-\sin 2x=0$

$\sin 3x-\sin 2x=0$

$2\sin\frac{x}{2}\cos\frac{5x}{2}=0$

$\sin\frac{x}{2}=0 \lor \cos\frac{5x}{2}=0$

$\frac{x}{2}=k\pi$ $\qquad$ $\frac{5x}{2}=\frac{\pi}{2}+k\pi$

$x=2k\pi$ $\qquad$ $5x=\pi+2k\pi$

$x=\frac{\pi}{5}+\frac{2k\pi}{5}$

d) $\sin x + \cos x = \sqrt{2}\sin 5x$

$\sin x + \sin\left(\frac{\pi}{2} - x\right) = \sqrt{2}\sin 5x$

$2\sin\frac{x + \frac{\pi}{2} - x}{2}\cos\frac{x - \frac{\pi}{2} + x}{2} = \sqrt{2}\sin 5x$

$2\sin\frac{\pi}{4}\cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}\sin 5x$

$2\cdot\frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{2} - x + \frac{\pi}{4}\right) = \sqrt{2}\sin 5x$

$\sqrt{2}\sin\left(\frac{3\pi}{4} - x\right) = \sqrt{2}\sin 5x \quad | : \sqrt{2}$

$\sin 5x - \sin\left(\frac{3\pi}{4} - x\right) = 0$

$2\sin\frac{5x - \frac{3\pi}{4} + x}{2}\cos\frac{5x + \frac{3\pi}{4} - x}{2} = 0$

$2\sin\left(3x - \frac{3\pi}{8}\right)\cos\left(2x + \frac{3\pi}{8}\right) = 0$

$\sin\left(3x - \frac{3\pi}{8}\right) = 0 \quad \vee \quad \cos\left(2x + \frac{3\pi}{8}\right) = 0$

$3x - \frac{3\pi}{8} = k\pi \qquad 2x + \frac{3\pi}{8} = \frac{\pi}{2} + k\pi$

$3x = \frac{3\pi}{8} + k\pi \qquad 2x = \frac{\pi}{2} - \frac{3\pi}{8} + k\pi$

$x = \frac{\pi}{8} + \frac{k\pi}{3} \qquad 2x = \frac{\pi}{8} + k\pi$

$x = \frac{\pi}{16} + \frac{k\pi}{2}, \quad k \in \mathbb{Z}$

e) $\sin 5x + \sin x + 2\sin^2 x = 1$

$2\sin 3x\cos 2x = 1 - 2\sin^2 x$

$2\sin 3x\cos 2x = \cos 2x$

$2\sin 3x\cos 2x - \cos 2x = 0$

$\cos 2x\left(2\sin 3x - 1\right) = 0$

$\cos 2x = 0 \quad \vee \quad 2\sin 3x - 1 = 0$

$2x = \frac{\pi}{2} + k\pi \qquad \sin 3x = \frac{1}{2}$

$x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad 3x = \frac{\pi}{6} + 2k\pi \qquad 3x = \frac{5\pi}{6} + 2k\pi$

$x = \frac{\pi}{18} + \frac{2k\pi}{3} \qquad x = \frac{5\pi}{18} + \frac{2k\pi}{3}, \quad k \in \mathbb{Z}$

4.3. Trigonometrijske nejednačine

$\sin x,\ \cos x \to 2\pi \to [0, 2\pi), (-\pi, \pi], [\frac{\pi}{2}, \frac{5\pi}{2}), [-\frac{\pi}{2}, \frac{3\pi}{2}) + 2k\pi, k\in\mathbb{Z}$

$\operatorname{tg} x \to \pi \to (-\frac{\pi}{2}, \frac{\pi}{2}] + k\pi, k\in\mathbb{Z}$

25. a) $\sin x > 0$

Nad intervalom $[0, 2\pi)$, rešenje je interval $(0, \pi)$, a nad skupom realnih brojeva rešenje je unija svih intervala koji se dobijaju translacijom ovog intervala za $2k\pi$, $k\in\mathbb{Z}$.

$x \in \bigcup_{k\in\mathbb{Z}} (2k\pi, \pi + 2k\pi)$

b) $\sin x > \frac{1}{2}$

Nad intervalom $[0, 2\pi)$ rešenje je interval $(\frac{\pi}{6}, \frac{5\pi}{6})$, a nad skupom R rešenje je unija svih intervala $(2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6})$, $k\in\mathbb{Z}$.

$x \in \bigcup_{k\in\mathbb{Z}} (2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6})$.

c) $\cos x < -\frac{\sqrt{2}}{2}$

$x \in \bigcup_{k\in\mathbb{Z}} (\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi)$

d) $\sin x + \sqrt{3}\cos x > 0$

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x > 0$

$\sin x \cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos x > 0$

$\sin(x + \frac{\pi}{3}) > 0$

$x + \frac{\pi}{3} \in \bigcup_{k\in\mathbb{Z}} (2k\pi, \pi + 2k\pi)$

$x \in \bigcup_{k\in\mathbb{Z}} (-\frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi)$

e) $2\cos^2 x + 5\cos x + 2 \geq 0$ $\quad \cos x = t$ $\quad (-\pi, \pi]$

$2t^2 + 5t + 2 \geq 0$

$t_{1,2} = \dfrac{-5 \pm \sqrt{25-16}}{4} \nearrow t_1 = -\frac{1}{2}, \searrow t_2 = -2$

$\cos x \in (-\infty, -\frac{1}{2}) \cup (2, +\infty)$

$-1 \leq \cos x \leq 1$

$-\frac{1}{2} \leq \cos x \leq 1$

$x \in \bigcup\limits_{k \in \mathbb{Z}} \left(-\frac{2\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi\right)$

f) $\operatorname{tg} x + \sqrt{3} < 0$

$\operatorname{tg} x < -\sqrt{3}$

$x \in \bigcup\limits_{k \in \mathbb{Z}} \left(-\frac{\pi}{2} + k\pi, -\frac{\pi}{3} + k\pi\right)$

26. $\log_2\left(\sin \frac{x}{2}\right) < -1$ $\quad (0, 2\pi)$

$0 < \sin \frac{x}{2} < \frac{1}{2}$

$0 < \frac{x}{2} < \frac{\pi}{6} \quad \vee \quad \frac{5\pi}{6} < \frac{x}{2} < \pi$

$x \in \left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$

4.4. Zadaci za vežbu

1. $\sin x = ?$ $\quad 4\cos x + 3\sin x = 5$

$\cos x = ?$ $\quad \sin^2 x + \cos^2 x = 1$

$\sin x = \dfrac{5 - 4\cos x}{3}$

$$\left(\frac{25-40\cos x+16\cos^2 x}{9}\right)+\cos^2 x=1$$

$$25\cos^2 x-40\cos x+25=9$$

$$25\cos^2 x-40\cos x+16=0$$

$$\cos x_{1,2}=\frac{40\pm\sqrt{1600-1600}}{50}=\frac{4}{5}$$

$$\sin x=\frac{3}{5}$$

2. $4\sin^2 x\,\frac{3x+\pi}{6}=3$

$$\sin^2 x\,\frac{3x+\pi}{6}=\frac{3}{4}$$

$$\sin x\,\frac{3x+\pi}{6}=\pm\frac{\sqrt{3}}{2}$$

$$\frac{3x+\pi}{6}=\frac{\pi}{3}+2k\pi \qquad \frac{3x+\pi}{6}=\frac{2\pi}{3}+2k\pi$$

$$3x+\pi=2\pi+12k\pi \qquad 3x+\pi=4\pi+12k\pi$$

$$3x=\pi+12k\pi \qquad 3x=3\pi+12k\pi$$

$$x=\frac{\pi}{3}+4k\pi \qquad x=\pi+4k\pi$$

$$x=\frac{\pi}{3}+2k\pi \qquad x=\pi+2k\pi=(2k+1)\pi$$

$$\frac{3x+\pi}{6}=\frac{4\pi}{3}+2k\pi \qquad \frac{3x+\pi}{6}=\frac{5\pi}{3}+2k\pi$$

$$3x+\pi=8\pi+12k\pi \qquad 3x+\pi=10\pi+12k\pi$$

$$3x=7\pi+12k\pi \qquad 3x=9\pi+12k\pi$$

$$x=\frac{7\pi}{3}+4k\pi \qquad x=3\pi+4k\pi$$

$$x=\frac{7\pi}{3}+2k\pi=\frac{\pi}{3}+(2k+1)\pi \qquad x=3\pi+2k\pi$$

$$\left[x=\frac{\pi}{3}+2k\pi \ \vee\ x=(2k+1)\pi,\ k\in\mathbb{Z}\right]$$

3. $ctg x = tg x$

$\frac{1}{tg x} - tg x = 0 \quad / \cdot tg x$

$1 - tg^2 x = 0$

$(1 - tg x)(1 + tg x) = 0$

$1 - tg x = 0 \quad \vee \quad 1 + tg x = 0$

$tg x = 1 \qquad tg x = -1$

$x = \frac{\pi}{4} + k\pi \qquad x = \frac{3\pi}{4} + k\pi$

$x = \frac{\pi}{4} + \frac{2k\pi}{2} \qquad x = \frac{\pi}{4} + \frac{2\pi}{4} + \frac{2k\pi}{2}$

$x = \frac{\pi}{4} + \frac{\pi}{2} + \frac{2k\pi}{2}$

$x = \frac{\pi}{4} + \frac{(2k+1)\pi}{2}$

Rešenje: $x = \frac{\pi}{4} + \frac{k\pi}{2}$

$x = \frac{\pi + 2k\pi}{4}$

$x = \frac{(2k+1)\pi}{4}, \quad k \in Z$

4. $ctg\left(\frac{5\pi}{12} - x\right) + 1 = 0$

$ctg\left(\frac{5\pi}{12} - x\right) = -1$

$\frac{5\pi}{12} - x = \frac{3\pi}{4} + k\pi$

$x = \frac{5\pi}{12} - \frac{3\pi}{4} + k\pi$

$x = -\frac{4\pi}{12} + k\pi$

$x = -\frac{\pi}{3} + k\pi, \quad k \in Z$

5. $2\sin^2 2x - 1 = 0$

$2\sin^2 2x = 1$

$\sin^2 2x = \frac{1}{2}$

$\sin 2x = \pm\frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + 2k\pi \qquad 2x = \frac{3\pi}{4} + 2k\pi$

$x = \frac{\pi}{8} + k\pi \qquad x = \frac{3\pi}{8} + k\pi$

$2x = \frac{5\pi}{4} + 2k\pi \qquad 2x = \frac{7\pi}{4} + 2k\pi$

$x = \frac{5\pi}{8} + k\pi \qquad x = \frac{7\pi}{8} + k\pi$

$$x=\frac{\pi}{8}+k\frac{\pi}{4}$$
$$x=\frac{\pi+2k\pi}{8}$$
$$x=\frac{(2k+1)\pi}{8}$$

6. $2\sin\frac{x}{2}=1-\cos x$

$$2\sin\frac{x}{2}=2\sin^2\frac{x}{2}$$
$$2\sin^2\frac{x}{2}-2\sin\frac{x}{2}=0$$
$$2\sin\frac{x}{2}\left(\sin\frac{x}{2}-1\right)=0$$
$$\sin\frac{x}{2}=0 \quad \vee \quad \sin\frac{x}{2}-1=0$$
$$\frac{x}{2}=k\pi \qquad \sin\frac{x}{2}=1$$
$$x=2k\pi \qquad \frac{x}{2}=\frac{\pi}{2}+2k\pi$$
$$x=\pi+4k\pi$$
$$x=(4k+1)\pi$$

7. $tg\,x=\frac{\cos x-\sin x}{\cos x+\sin x}$

$$\frac{\sin x}{\cos x}=\frac{\cos x-\sin x}{\cos x+\sin x}$$
$$\sin^2 x+\sin x\cos x=\cos^2 x-\sin x\cos x$$
$$\cos^2 x-\sin^2 x=2\sin x\cos x$$
$$\cos 2x=\sin 2x \quad | : \cos 2x$$
$$tg\,2x=1$$
$$2x=\frac{\pi}{4}+k\pi$$
$$x=\frac{\pi}{8}+\frac{k\pi}{2},\ k\in Z$$

8. $4\sin\frac{x}{2}\cos\frac{x}{2} = 1$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} + 2k\pi \quad x = \frac{5\pi}{6} + 2k\pi,\ k \in Z$

9. $\sqrt{3}\cos x + \sin x = 2 \quad |:2$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 1$

$\sin\left(\frac{\pi}{3} + x\right) = 1$

$\frac{\pi}{3} + x = \frac{\pi}{2} + 2k\pi$

$x = \frac{\pi}{2} - \frac{\pi}{3} + 2k\pi$

$x = \frac{\pi}{6} + 2k\pi,\ k \in Z$

10. $\sqrt{3}\sin x - \cos x = -1 \quad |:2$

$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = -\frac{1}{2}$

$\sin\left(x - \frac{\pi}{6}\right) = -\frac{1}{2}$

$x - \frac{\pi}{6} = \frac{7\pi}{6} + 2k\pi \qquad x - \frac{\pi}{6} = -\frac{\pi}{6} + 2k\pi$

$x = \frac{4\pi}{3} + 2k\pi \qquad x = 2k\pi,\ k \in Z$

11. $(2\sqrt{2} - 2)\cos t = 4\sin^2 t + \sqrt{2} - 4$

$(2\sqrt{2} - 2)\cos t = -4(1 - \sin^2 t) + \sqrt{2}$

$(2\sqrt{2} - 2)\cos t = \sqrt{2} - 4\cos^2 t$

$2\sqrt{2}\cos t - 2\cos t - \sqrt{2} + 4\cos^2 t = 0$

$\sqrt{2}(2\cos t - 1) + 2\cos t(2\cos t - 1) = 0$

$(2\cos t + \sqrt{2})(2\cos t - 1) = 0$

$2\cos t + \sqrt{2} = 0 \quad \vee \quad 2\cos t - 1 = 0$

$2\cos t = -\sqrt{2} \qquad 2\cos t = 1$

$\cos t = -\frac{\sqrt{2}}{2} \qquad \cos t = \frac{1}{2}$

$\cos t = \pm\frac{3\pi}{4} + 2k\pi \qquad \cos t = \pm\frac{\pi}{3} + 2k\pi, \ k \in \mathbb{Z}$

12. $\sin^4 x + \cos^4 x = \cos 4x$

$\sin^4 x + \cos^4 x = \cos^2 2x - \sin^2 2x$

$\sin^4 x + \cos^4 x = (\cos^2 x - \sin^2 x)^2 - 4\sin^2 x\cos^2 x$

$\cancel{\sin^4 x} + \cancel{\cos^4 x} = \cancel{\cos^4 x} - 2\cos^2 x\sin^2 x + \cancel{\sin^4 x} - 4\sin^2 x\cos^2 x$

$6\cos^2 x\sin^2 x = 0$

$\cos^2 x = 0 \quad \vee \quad \sin^2 x = 0 \qquad R: x = \frac{k\pi}{2}$

$\cos x = 0 \qquad \sin x = 0$

$x = \frac{\pi}{2} + k\pi \qquad x = k\pi$

13. $3\sin\left(\frac{\pi}{2} - x\right) - 4\sin(\pi + x)\sin\left(\frac{5\pi}{2} + x\right) + 8\cos^2\frac{x}{2} = 4$

$3\cos x + 4\sin x\cos x + 4(1 + \cos x) = 4$

$3\cos x + 4\sin x\cos x + \cancel{4} + 4\cos x = \cancel{4}$

$7\cos x + 4\sin x\cos x = 0$

$\cos x(7 + 4\sin x) = 0$

$\cos x = 0 \quad \vee \quad 7 + 4\sin x = 0$

$x = \frac{\pi}{2} + k\pi \qquad 4\sin x = -7$

$\cancel{\sin x = \frac{-7}{4}} \quad -1 \le \sin x \le 1$

14. $\operatorname{tg} 6x - 3\operatorname{tg} 3x = 0$

$\frac{2\operatorname{tg} 3x}{1 - \operatorname{tg}^2 3x} - 3\operatorname{tg} 3x = 0 \qquad \operatorname{tg} 3x = t$

$\frac{2t}{1 - t^2} - 3t = 0 \quad | \cdot (1 - t^2)$

$$2t - 3t + 3t^3 = 0$$
$$3t^3 - t = 0$$
$$t(3t^2 - 1) = 0$$
$$t = 0 \qquad 3t^2 - 1 = 0$$
$$3t^2 = 1$$
$$t^2 = \frac{1}{3}$$
$$t = \frac{1}{\sqrt{3}}$$
$$t = \frac{\sqrt{3}}{3}$$

$$tg\,3x = 0$$
$$3x = k\pi$$
$$x = \frac{k\pi}{3}$$

$$tg\,3x = \frac{\sqrt{3}}{3}$$
$$3x = \frac{\pi}{6} + k\pi$$
$$x = \frac{\pi}{18} + \frac{k\pi}{3} \quad k \in \mathbb{Z}$$

15. $$2\cos 4x - 2\cos 2x = 4\cos^2 x - 1$$
$$2(2\cos^2 2x - 1) - 2\cos 2x = 4\cos^2 x - 1 - 1 + 1$$
$$4\cos^2 2x - 2 - 2\cos 2x = 2(2\cos^2 x - 1) + 1$$
$$4\cos^2 2x - 2\cos 2x - 2 = 2\cos 2x + 1$$
$$4\cos^2 2x - 4\cos 2x - 3 = 0$$
$$\cos 2x_{1,2} = \frac{4 \pm \overset{8}{\sqrt{16+48}}}{8} \begin{cases} \nearrow \cos 2x_1 = \cancel{\frac{3}{2}} \\ \searrow \cos 2x_2 = -\frac{1}{2} \end{cases}$$

$$\cos 2x = -\frac{1}{2}$$
$$2x = \frac{2\pi}{3} + 2k\pi \qquad 2x = \frac{4\pi}{3} + 2k\pi$$
$$x = \frac{2\pi}{6} + k\pi \qquad x = \frac{4\pi}{6} + k\pi$$
$$x = \frac{\pi}{3} + k\pi \qquad x = \frac{2\pi}{3} + k\pi$$

16. $\alpha = \frac{\pi}{3} \quad \beta = \frac{2\pi}{3}$

$$\frac{\sin^2(\alpha+\beta) - \cos^2\alpha - \cos^2\beta}{\sin(\alpha+\beta) - \sin^2\alpha - \sin^2\beta} =$$

$$= \frac{\sin^2\pi - \cos^2\alpha - \cos^2 2\alpha}{\sin\pi - \sin^2\alpha - \sin^2 2\alpha} =$$

$$= \frac{-\cos^2\alpha - (\cos^2\alpha - \sin^2\alpha)^2}{-\sin^2\alpha - (2\sin\alpha\cos\alpha)^2}$$

$$= \frac{-\cos^2\alpha - \cos^4\alpha + 2\sin^2\alpha\cos^2\alpha - \sin^4\alpha}{-\sin^2\alpha - 4\sin^2\alpha\cos^2\alpha} =$$

$$= \frac{-\frac{1}{4} - \frac{1}{16} + \cancel{2}\cdot\frac{3}{\cancel{4}_2}\cdot\frac{1}{4} - \frac{9}{16}}{-\frac{3}{4} - \cancel{4}\cdot\frac{3}{\cancel{4}}\cdot\frac{1}{4}} =$$

$$= \frac{-\frac{4}{16} - \frac{1}{16} + \frac{6}{16} - \frac{9}{16}}{-\frac{6}{4}} =$$

$$= \frac{-\frac{2}{\cancel{4}}}{-\frac{6}{\cancel{4}}} = \frac{2}{6} = \frac{1}{3}$$

17. $\cos\left(\frac{\pi}{4} - x\right) = \sin\left(\frac{\pi}{6} + x\right)$

$$\cos\left(\frac{\pi}{4} - x\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{6} - \frac{\pi}{2} + x\right)$$

$$\cos\left(\frac{\pi}{4} - x\right) = \sin\left(\frac{\pi}{2} + x - \frac{\pi}{3}\right)$$

$$\cos\left(\frac{\pi}{4} - x\right) = \cos\left(x - \frac{\pi}{3}\right)$$

$$\frac{\pi}{4} - x = x - \frac{\pi}{3} \quad \vee \quad \cancel{\frac{\pi}{4} - x = \frac{\pi}{3} - x}$$

$$\frac{\pi}{4} + \frac{\pi}{3} = 2x + 2k\pi$$

$$2x = \frac{7\pi}{12} + 2k\pi$$

$$x = \frac{7\pi}{24} + k\pi$$

18. $\left(\frac{\cos x+1}{\sin x}\right)^2=\frac{1}{3}$

$$\frac{\cos^2 x+2\cos x+1}{\sin^2 x}-\frac{1}{3}=0$$

$$\frac{3\cos^2 x+6\cos x+3-\sin^2 x}{3\sin^2 x}=0$$

$$\frac{3\cos^2 x+6\cos x+3-1+\cos^2 x}{3\sin^2 x}=0$$

$$\frac{4\cos^2 x+6\cos x+2}{3\sin^2 x}=0 \qquad \cos x=t$$

$$4t^2+6t+2=0$$

$$t_{1,2}=\frac{-6\pm\sqrt{36-32}}{8} \to t_1=-\frac{1}{2}$$

$$\to \cancel{t_2=-1} \Rightarrow \sin x\neq 0$$

$$\cos x=-\frac{1}{2}$$

$$x=\pm\frac{2\pi}{3}+2k\pi,\ k\in\mathbb{Z}$$

19. $\sin\left(\frac{\pi}{2}+\frac{x}{2}\right)=\cos(\pi-x)-1$

$$\cos\frac{x}{2}=-\cos x-1$$

$$\cos\frac{x}{2}=-2\cos^2\frac{x}{2}$$

$$2\cos^2\frac{x}{2}+\cos\frac{x}{2}=0$$

$$\cos\frac{x}{2}\left(2\cos\frac{x}{2}+1\right)=0$$

$$\cos\frac{x}{2}=0 \quad \vee \quad 2\cos\frac{x}{2}+1=0$$

$$\frac{x}{2}=\frac{\pi}{2}+k\pi \qquad 2\cos\frac{x}{2}=-1$$

$$x=\pi+2k\pi \qquad \cos\frac{x}{2}=-\frac{1}{2}$$

$$x=(2k+1)\pi \qquad \frac{x}{2}=\pm\frac{2\pi}{3}+2k\pi$$

$$x=\pm\frac{4\pi}{3}+4k\pi,\ k\in\mathbb{Z}$$

20. $1+2\cos^2\left(x+\frac{\pi}{6}\right)=3\sin\left(\frac{\pi}{3}-x\right)$

$1+2\cos^2\left(x+\frac{\pi}{6}\right)=3\cos\left(\frac{\pi}{2}-\frac{\pi}{3}+x\right)$

$1+2\cos^2\left(x+\frac{\pi}{6}\right)=3\cos\left(x+\frac{\pi}{6}\right)$ $\qquad \cos\left(x+\frac{\pi}{6}\right)=t$

$2t^2-3t+1=0$

$t_{1,2}=\frac{3\pm\sqrt{9-8}}{4}$ $\rightarrow t_1=1$, $\rightarrow t_2=\frac{1}{2}$

$\cos\left(x+\frac{\pi}{6}\right)=1 \quad \vee \quad \cos\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$

$x+\frac{\pi}{6}=2k\pi$

$x=-\frac{\pi}{6}+2k\pi$

$x+\frac{\pi}{6}=\frac{\pi}{3}+2k\pi \quad x+\frac{\pi}{6}=-\frac{\pi}{3}+2k\pi$

$x=\frac{\pi}{6}+2k\pi \quad x=-\frac{\pi}{2}+2k\pi, \ k\in \mathbb{Z}$

21. $\sqrt{3}\sin 3x=\cos 3x+1 \quad |:2$

$\frac{\sqrt{3}}{2}\sin 3x-\frac{1}{2}\cos 3x=\frac{1}{2}$

$\sin\left(3x-\frac{\pi}{6}\right)=\frac{1}{2}$

$3x-\frac{\pi}{6}=\frac{\pi}{6}+2k\pi \quad \vee \quad 3x-\frac{\pi}{6}=\frac{5\pi}{6}+2k\pi$

$3x=\frac{\pi}{3}+2k\pi \qquad 3x=\pi+2k\pi$

$x=\frac{\pi}{9}+\frac{2k\pi}{3} \qquad x=\frac{\pi}{3}+\frac{2k\pi}{3}, \ k\in \mathbb{Z}$

22. $\sin 3x+\sin 2x+\sin x=0$

$\sin 2x+2\sin 2x\cos x=0$

$\sin 2x(1+2\cos x)=0$

$\sin 2x=0 \quad \vee \quad 1+2\cos x=0$

$2x=k\pi \qquad 2\cos x=-1$

$x=\frac{k\pi}{2} \qquad \cos x=-\frac{1}{2}$

$x=\pm\frac{2\pi}{3}-2k\pi, \ k\in \mathbb{Z}$

23. $\sin x + \sin 9x = \sqrt{2}\cos 4x$

$2\sin 5x \cos 4x - \sqrt{2}\cos 4x = 0$

$\cos 4x\left(2\sin 5x - \sqrt{2}\right) = 0$

$\cos 4x = 0 \quad \vee \quad 2\sin 5x - \sqrt{2} = 0$

$4x = \frac{\pi}{2} + k\pi \qquad 2\sin 5x = \sqrt{2}$

$x = \frac{\pi}{8} + \frac{k\pi}{4} \qquad \sin 5x = \frac{\sqrt{2}}{2}$

$5x = \frac{\pi}{4} + 2k\pi \qquad 5x = \frac{3\pi}{4} + 2k\pi$

$x = \frac{\pi}{20} + \frac{2k\pi}{5} \qquad x = \frac{3\pi}{20} + \frac{2k\pi}{5}, \ k \in \mathbb{Z}$

24. $\alpha \in [-\pi, \pi]$

$2x^2 + 2x + \cos\alpha = 0$

$D < 0$

$4 - 8\cos\alpha < 0$

$4(1 - 2\cos\alpha) < 0$

$1 - 2\cos\alpha < 0$

$-2\cos\alpha < -1$

$2\cos\alpha > 1$

$\cos\alpha > \frac{1}{2}$

$\alpha \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

25. $\sin x \leq -\frac{\sqrt{3}}{2} \qquad [0, 2\pi)$

$x \in \bigcup_{k \in \mathbb{Z}} \left(\frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi\right)$

26. $\cos 2x > 0$

$2x \in \bigcup_{k \in \mathbb{Z}} \left(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right)$

$x \in \bigcup_{k \in \mathbb{Z}} \left(-\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi\right)$

27. $\sin 3x < \sin x$

$\sin 3x - \sin x < 0$

$2\sin x \cos 2x < 0$

$2\sin x\,(1 - 2\sin^2 x) < 0$

$\sin x < 0 \quad \vee \quad 1 - 2\sin^2 x < 0$

$x \in (\pi, 2\pi) \qquad 2\sin^2 x > 1$

$\sin^2 x > \frac{1}{2}$

$\sin x > \frac{\sqrt{2}}{2} \qquad \sin x < -\frac{\sqrt{2}}{2}$

$x \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \qquad x \in \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

	$\frac{\pi}{4}$ – $\frac{3\pi}{4}$	$\frac{3\pi}{4}$ – π	π – $\frac{5\pi}{4}$	$\frac{5\pi}{4}$ – $\frac{7\pi}{4}$	$\frac{7\pi}{4}$ – 2π
$2\sin x$	+	+	−	−	−
$1-2\sin^2 x$	−	+	+	−	+
	−	+	−	+	−

$x \in \bigcup_{k\in\mathbb{Z}} \left(\frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi\right) \cup$

$\cup \left(\pi + 2k\pi, \frac{5\pi}{4} + 2k\pi\right) \cup$

$\cup \left(\frac{7\pi}{4} + 2k\pi, 2\pi + 2k\pi\right)$

28. $tg^2 x - (1 + \sqrt{3})\,tg x + \sqrt{3} < 0$

$tg^2 x - tg x - \sqrt{3}\,tg x + \sqrt{3} < 0$

$tg x\,(tg x - 1) - \sqrt{3}\,(tg x - 1) < 0$

$(tg x - \sqrt{3})(tg x - 1) < 0$

$tg x - \sqrt{3} < 0 \qquad tg x - 1 > 0$

$tg x < \sqrt{3} \qquad tg x > 1$

$1 < tg x < \sqrt{3}$

$x \in \bigcup_{k\in\mathbb{Z}} \left(\frac{\pi}{4} + k\pi, \frac{\pi}{3} + k\pi\right)$

29. $\sin x \le -1$

$x \in \left\{ \frac{3\pi}{2} + 2k\pi \;\middle|\; k \in \mathbb{Z} \right\}$

30. $\sin x - \sqrt{3}\sin 3x + \sin 5x < 0 \qquad x \in (0, \pi)$

$2\sin 3x \cos 2x - \sqrt{3}\sin 3x < 0$

$\sin 3x\,(2\cos 2x - \sqrt{3}) < 0$

$\sin 3x < 0$

$3x \in (\pi, 2\pi)$

$x \in \left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$

$2\cos 2x - \sqrt{3} < 0$

$2\cos 2x < \sqrt{3}$

$\cos 2x < \frac{\sqrt{3}}{2}$

$2x \in \left(\frac{\pi}{6}, \frac{11\pi}{6}\right)$

$x \in \left(\frac{\pi}{12}, \frac{11\pi}{12}\right)$

	$\frac{\pi}{12}$ – $\frac{\pi}{3}$	$\frac{\pi}{3}$ – $\frac{2\pi}{3}$	$\frac{2\pi}{3}$ – $\frac{11\pi}{12}$
$\sin 3x$	+	−	+
$2\cos 2x - \sqrt{3}$	−	−	−
$\sin 3x(2\cos 2x - \sqrt{3})$	−	+	−

$x \in \left(\frac{\pi}{12}, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{11\pi}{12}\right)$

33. Vidi 28. :)

37. $\operatorname{tg}x(2 - \sin x) = \frac{3}{4\cos x}$

$2\operatorname{tg}x - \frac{\sin x}{\cos x} \cdot \sin x = \frac{3}{4\cos x}$

$2\frac{\sin x}{\cos x} - \frac{\sin^2 x}{\cos x} = \frac{3}{4\cos x}$

$\frac{2\sin x - \sin^2 x}{\cos x} = \frac{3}{4\cos x} \quad / \cdot 4\cos x$

$-4\sin^2 x + 8\sin x - 3 = 0 \qquad \sin x = t$

$-4t^2 + 8t - 3 = 0$

$t_{1,2} = \frac{-8 \pm \sqrt{64 - 48}}{-8} \rightarrow t_1 = \frac{1}{2}, \quad \cancel{t_2 = \frac{3}{2}}$

$\sin x = \frac{1}{2}$

$x \in \left\{\frac{\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\right\} \cup \left\{\frac{5\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\right\}$