29. $a \in R \quad b \in R$

$ax^2 - x + b = 0$

$x = x_0$

$2x_0 = x_0^2$

$D = 0$

$1 - 4ab = 0$

$4ab = 1$

$\boxed{ab = \frac{1}{4}}$

$-\frac{-1}{a} = \frac{b}{a}$

$\frac{1}{a} = \frac{b}{a}$

$a = ab$

$a(b-1) = 0$

$a_2 = 0 \quad b - 1 = 0$

$b_1 = 1 \quad a_1 = \frac{1}{4}$

3. Eksponencijalni izrazi i logaritmi

3.1. Eksponencijalne jednačine i nejednačine

↙ eksponencijalna jednačina

$a^x = b \quad a, b \in R \quad a, b > 0 \quad a \neq 1$

$y = a^x$, $a > 0$ i $a \neq 1 \Rightarrow$ eksponencijalna funkcija

$a > 1$ – funkcija je monotono rastuća, pa je

$a^{f(x)} \geq a^{g(x)} \Leftrightarrow f(x) \geq g(x)$

$0 < a < 1 \Rightarrow$ funkcija je monotono opadajuća, pa je

$a^{f(x)} \geq a^{g(x)} \Leftrightarrow f(x) \leq g(x)$

* Za eksponencijalnu funkciju važi:

$a^x > 0 \ \forall x \in R \quad a^x = a^y \Leftrightarrow x = y$

Specijalno je: $a^x = 1 \Leftrightarrow x = 0$

1. a) $3^x - 2\cdot 3^{x-1} = \frac{1}{9}$

$3^x - 2\cdot 3^x \cdot 3^{-1} = \frac{1}{3^2}$

$3^x\left(1-\frac{2}{3}\right) = \frac{1}{3^2}$

$3^x \cdot \frac{1}{3} = \frac{1}{3^2}$

$3^x = \frac{\frac{1}{3^2}}{\frac{1}{3}}$

$3^x = 3^{-1}$

$x = -1$.

b) $\left(\frac{1}{2}\right)^{x^2-20x+61,5} = \frac{8}{\sqrt{2}}$

$\left(\frac{1}{2}\right)^{x^2-20x+61,5} = \frac{2^3}{2^{\frac{1}{2}}}$

$\left(\frac{1}{2}\right)^{x^2-20x+61,5} = 2^{\frac{5}{2}}$

$\left(\frac{1}{2}\right)^{x^2-20x+61,5} = \left(\frac{1}{2}\right)^{-\frac{5}{2}}$

$x^2-20x+61,5 = -\frac{5}{2} \quad |\cdot 2$

$2x^2-40x+123 = -5$

$2x^2-40x+128 = 0 \quad |:2$

$x^2-20x+64 = 0$

$x_{1,2} = \frac{20 \pm \sqrt{400-256}}{2} \rightarrow x_1 = 4, \quad x_2 = 16$

c) $3^{\sqrt{x}} + 2\cdot 3^{\sqrt{x}-1} = 15$

$3^{\sqrt{x}} + 2\cdot 3^{\sqrt{x}}\cdot 3^{-1} = 15$

$3^{\sqrt{x}}\left(1+\frac{2}{3}\right) = 15$

$3^{\sqrt{x}}\cdot \frac{5}{3} = 15$

$3^{\sqrt{x}} = \frac{\frac{15}{1}}{\frac{5}{3}}$

$3^{\sqrt{x}} = 3^2$

$\sqrt{x} = 2$

$x = 4$

d) $\frac{1}{8}\cdot 4^{2x-3} = \left(\frac{\sqrt{2}}{8}\right)^{-x}$

$\frac{1}{2^3}\cdot 2^{4x-6} = \left(\frac{2^{\frac{1}{2}}}{2^3}\right)^{-x}$

$2^{4x-9} = 2^{-\frac{5}{2}(-x)}$

$4x-9 = \frac{5}{2}x$

$8x-18 = 5x$

$3x = 18$

$x = 6$

2. a) $3^x - 3\cdot 2^y = -11$

$4\cdot 3^x + 2^y = 8$

$3^x = a$

$2^y = b$

$a - 3b = -11$

$4a + b = 8 \quad |\cdot 3$

$a - 3b = -11$

$12a + 3b = 24$

$13a = 13$

$a = 1$

$b = 4$

$3^x = 1$

$x = 0$

$2^y = 4$

$y = 2$

b) $2 \cdot 3^{x+1} - 2^{y+1} = 17$ $\quad 3^x = a$

$3^x + 2^{y+1} = 4$ $\quad 2^{y+1} = b$

$2 \cdot a \cdot 3 - b = 17$

$a + b = 4$

$6a - b = 17$

$a + b = 4$

$7a = 21$

$a = 3$

$b = 1$

$3^x = 3$

$x = 1$

$2^{y+1} = 1$

$y + 1 = 0$

$y = -1$

3. a) $16^x - 3 \cdot 4^x + 2 = 0$

$4^{2x} - 3 \cdot 4^x + 2 = 0$ $\quad 4^x = t$

$t^2 - 3t + 2 = 0$

$t_{1,2} = \frac{3 \pm \sqrt{9-8}}{2}$ $\to t_1 = 2$, $\to t_2 = 1$

$4^x = 2$

$2^{2x} = 2$

$2x = 1$

$x_1 = \frac{1}{2}$

$4^x = 1$

$x_2 = 0$

b) $4^{\sqrt{x-2}} + 16 = 10 \cdot 2^{\sqrt{x-2}}$

$2^{2\sqrt{x-2}} - 10 \cdot 2^{\sqrt{x-2}} + 16 = 0$ $\quad 2^{\sqrt{x-2}} = t$

$t^2 - 10t + 16 = 0$

$t_{1,2} = \frac{10 \pm \sqrt{100-64}}{2}$ $\to x_1 = 8$, $\to x_2 = 2$

$2^{\sqrt{x-2}} = 8$

$2^{\sqrt{x-2}} = 2^3$

$\sqrt{x-2} = 3$

$x - 2 = 9$

$x = 11$

$2^{\sqrt{x-2}} = 2$

$\sqrt{x-2} = 1$

$x - 2 = 1$

$x = 3$

c) $5^{2x-3} = 3 + 2 \cdot 5^{x-2}$

$(5^x)^2 \cdot 5^{-3} = 3 + 2 \cdot 5^x \cdot 5^{-2}$ $\quad 5^x = t$

$t^2 \cdot 5^{-3} - 3 - 2t \cdot 5^{-2} = 0$

$\frac{t^2}{5^3} - \frac{2t}{5^2} - 3 = 0 \quad | \cdot 125$

$t^2 - 10t - 375 = 0$

$t_{1,2} = \frac{10 \pm \sqrt{100+1500}}{2}$ $\to t_1 = 25$, $\to \cancel{t_2 = -15}$

$5^x = 25$

$5^x = 5^2$

$x = 2$

4. a) $20^x - 6 \cdot 5^x + 10^x = 0$

$(4 \cdot 5)^x - 6 \cdot 5^x + (2 \cdot 5)^x = 0$

$5^x (4^x - 6 + 2^x) = 0 \quad | : 5^x$

$2^{2x} + 2^x - 6 = 0 \qquad t = 2^x \qquad 2^x = 2$

$t^2 + t - 6 = 0 \qquad x = 1$

$t_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} \rightarrow t_1 = 2$, $\rightarrow \cancel{t_2 = -3}$

b) $7 \cdot 9^x - 10 \cdot 21^x + 3 \cdot 49^x = 0$

$7 \cdot 3^{2x} - 10 \cdot (7 \cdot 3)^x + 3 \cdot 7^{2x} = 0 \quad | : 7^x \cdot 3^x$

$7 \cdot \frac{3^x}{7^x} - 10 + 3 \cdot \frac{7^x}{3^x} = 0 \qquad \frac{3^x}{7^x} = t \qquad \frac{3^x}{7^x} = 1 \qquad \frac{3^x}{7^x} = \frac{3}{7}$

$x = 0 \qquad x = 1$

$7t - 10 + 3 \frac{1}{t} = 0 \quad | \cdot t$

$7t^2 - 10t + 3 = 0$

$t_{1,2} = \frac{10 \pm \sqrt{100-84}}{14} \rightarrow t_1 = 1$, $\rightarrow t_2 = \frac{6}{14} = \frac{3}{7}$

c) $2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$

$2 \cdot 2^{2x} - 5 \cdot (2 \cdot 3)^x + 3 \cdot 3^{2x} = 0 \quad | : 2^x \cdot 3^x$

$2 \cdot \frac{2^x}{3^x} - 5 + 3 \frac{3^x}{2^x} = 0 \qquad \frac{2^x}{3^x} = t \qquad \frac{2^x}{3^x} = \frac{3}{2} \qquad \frac{2^x}{3^x} = 1$

$2t - 5 + 3 \frac{1}{t} = 0 \quad | \cdot t \qquad x = -1 \qquad x = 0$

$2t^2 - 5t + 3 = 0$

$t_{1,2} = \frac{5 \pm \sqrt{25-24}}{4} \rightarrow x_1 = \frac{3}{2}$, $\rightarrow x_2 = 1$

5. a) $2^{3-6x} > 1$

$2^{3-6x} > 2^0$

$3 - 6x > 0$

$6x < 3$

$x < \frac{1}{2}$

b) $16^x > 0,125$

$2^{4x} > \frac{1}{8}$

$2^{4x} > \frac{1}{2^3}$

$2^{4x} > 2^{-3}$

$4x > -3$

$x > -\frac{3}{4}$

c) $(0,1)^{4x^2-2x-2} \leq (0,1)^{2x-3}$

$4x^2-2x-2 \geq 2x-3$

$4x^2-4x+1 \geq 0$

$x_{1,2} = \frac{4 \pm \sqrt{16-16}}{8} \quad x_{1,2} = \frac{1}{2}$

$x \in R$

d) $\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \frac{25}{4}$

$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{5}{2}\right)^{2}$

$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{2}{5}\right)^{-2}$

$\frac{6-5x}{2+5x} > -2$

$\frac{6-5x+4+10x}{2+5x} > 0$

$\frac{5x+10}{5x+2} > 0$

$5x+10>0 \qquad 5x+2>0$

$5x>-10 \qquad 5x>-2$

$x>-2 \qquad x>-\frac{2}{5}$

	$-\infty$ -2	$-\frac{2}{5}$	$+\infty$
$5x+10$	$-$	$+$	$+$
$5x+2$	$-$	$-$	$+$
	$+$	$-$	$+$

$x \in (-\infty,-2) \cup \left(-\frac{2}{5}, +\infty\right)$

3.2. Logaritmi

$a,b>0,\ a,b \in R,\ a \neq 1$

$a^x = b$

$x = \log_a b$

$x = 1 \rightarrow$ nula logaritamske funkcije $\rightarrow \log_a 1 = 0$

Za $a > 0$ logaritamska funkcija je monotono rastuća, pa je
$\log_a x \geq \log_a y \Leftrightarrow x \geq y$

Za $0 < a < 1$ logaritamska funkcija je monotono opadajuća, pa je
$\log_a x \geq \log_a y \Leftrightarrow x \leq y$

* Osobine logaritama za $x > 0$, $y > 0$, $a > 0$, $a \neq 1$:

$\log_a x = \log_a y \Leftrightarrow x = y$

$\log_a (x \cdot y) = \log_a x + \log_a y$

$\log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a x^n = n \log_a x$, $n \in N$

$\log_a \sqrt[n]{x} = \frac{1}{n} \log_a x$, $n \in N$

$\log_a 1 = 0$, $a^0 = 1$

$\log_a a = 1$, $a^1 = a$

$\log_a b = \frac{\log_c b}{\log_c a}$, $c > 0$, $c \neq 1$

$\log_a b = \frac{1}{\log_b a}$, $b > 0$, $b \neq 1$

$\log_{a^r} x = \frac{1}{r} \log_a x$, $r \neq 0$

$a^{y \log_a x} = (a^{\log_a x})^y = x^y$

$\log_a a^n = n$

6. a) $\log_{x-2}(2x-1)=2$

$x-2>0 \ \wedge \ x-2\neq 1 \ \wedge \ 2x-1>0$

$\underline{x>2}$ $\quad \underline{x\neq 3}$ $\quad 2x>1$

$x>\frac{1}{2}$

$x\in(2,3)\cup(3,+\infty)$

$(x-2)^2=2x-1$

$x^2-4x+4-2x-1=0$

$x^2-6x+5=0$

$x_{1,2}=\frac{6\pm\sqrt{36-20}}{2}$ $\nearrow x_1=5$ $\searrow \cancel{x_2=1}$ Rešenje: $x=5$

b) $\log_7(\log_5(\log_2 x))=0$, $x>0$

$\log_5(\log_2 x)=1$

$\log_2 x=5$

$x=2^5=32$

c) $\log_6(x+1)+\log_6(2x+1)=1$

$x+1>0 \ \wedge \ 2x+1>0$

$x>-1$ $\quad 2x>-1$

$\underline{x>-\frac{1}{2}}$

$\log_6(x+1)(2x+1)=1$

$\log_6(2x^2+x+2x+1)=1$

$\log_6(2x^2+3x+1)=1$

$2x^2+3x+1=6$

$2x^2+3x-5=0$

$x_{1,2}=\frac{-3\pm\sqrt{9+40}}{4}$ $\nearrow x_1=1$ $\searrow \cancel{x_2=-\frac{10}{4}}$ Rešenje: $x=1$

d) $(\log_5 x)^2 = 3 + \log_5 x^2, \quad x > 0$

$(\log_5 x)^2 - 2\log_5 x - 3 = 0 \qquad t = \log_5 x$

$t^2 - 2t - 3 = 0$

$t_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} \nearrow x_1 = 3, \ \searrow x_2 = -1$

$\log_5 x = 3 \qquad \log_5 x = -1$

$x = 5^3 = 125 \qquad x = 5^{-1} = \frac{1}{5}$

e) $\log_2(x^2 + 3x + 6) - \log_2 x = 3$

$\log_2 \frac{x^2 + 3x + 6}{x} = 3$

$\frac{x^2 + 3x + 6}{x} = 2^3 = 8 \quad | \cdot x$

$x^2 + 3x + 6 = 8x$

$x^2 - 5x + 6 = 0$

$x_{1,2} = \frac{5 \pm \sqrt{25 - 24}}{2} \nearrow x_1 = 3, \ \searrow x_2 = 2$

f) $\log_3 x^3 + (\log_3 x)^2 = 4, \quad x > 0$

$(\log_3 x)^2 + 3\log_3 x - 4 = 0 \qquad t = \log_3 x$

$t^2 + 3t - 4 = 0$

$t_{1,2} = \frac{-3 \pm \sqrt{9+16}}{2} \nearrow t_1 = 1, \ \searrow t_2 = -4$

$\log_3 x = 1 \qquad \log_3 x = -4$

$x = 3^1 = 3 \qquad x = 3^{-4} = \frac{1}{81}$

7. a) $\log_{x^3} 8 - \log_{\frac{1}{x^2}} 2 = 3$, $x > 0$, $x \neq 1$.

$\frac{1}{3}\log_x 2^3 - \log_{x^{-2}} 2 = 3$

$\frac{1}{3}\log_x 2^3 + \frac{1}{2}\log_x 2 = 3$

$\log_x 2 + \frac{1}{2}\log_x 2 = 3$

$\frac{3}{2}\log_x 2 = 3$

$\log_x 2 = \frac{\cancel{3}}{\frac{\cancel{3}}{2}}$

$\log_x 2 = 2$

$x^2 = 2$

$\underline{x_1 = \sqrt{2}}$ $\quad \cancel{x_2 = -\sqrt{2}}$ $\quad$ Rešenje: $x = \sqrt{2}$

b) $\log_{x^2} 81 - 3\log_{27} x = 1$

$\frac{1}{\log_{3^4} x^2} - 3\log_{3^3} x = 1$

$\frac{1}{\frac{1}{4}\log_3 x^2} - \log_3 x = 1$

$\frac{2}{\log_3 x} - \log_3 x = 1 \quad | \cdot \log_3 x$

$2 - (\log_3 x)^2 = \log_3 x$

$(\log_3 x)^2 + \log_3 x - 2 = 0 \qquad t = \log_3 x$

$t^2 + t - 2 = 0$

$t_{1,2} = \frac{-1 \pm \sqrt{1+8}^{\,3}}{2} \nearrow t_1 = 1, \searrow t_2 = -2$

$\log_3 x = 1 \qquad \log_3 x = -2$

$x = 3^1 = 3 \qquad x = 3^{-2} = \frac{1}{9}$

c) $\log_2 \sqrt[3]{x} - \log_4 x^3 + 2\log_8 x = \frac{3}{2}, \quad x > 0$

$\frac{1}{3}\log_2 x - 3\log_{2^2} x + 2\log_{2^3} x = \frac{3}{2}$

$\frac{1}{3}\log_2 x - \frac{3}{2}\log_2 x + \frac{2}{3}\log_2 x = \frac{3}{2}$

$-\frac{1}{2}\log_2 x = \frac{3}{2}$

$\log_2 x = -\frac{\frac{3}{2}}{\frac{1}{2}} = -3$

$x = 2^{-3} = \frac{1}{8}$

d) $\frac{1}{6}\log_2(x-2) - \frac{1}{3} = \log_{\frac{1}{8}} \sqrt{3x-5}$

$x - 2 > 0 \qquad \sqrt{3x-5} > 0$

$\underline{x > 2} \qquad 3x - 5 > 0$

$3x > 5$

$x > \frac{5}{3}$

$\frac{1}{6}\log_2(x-2) - \frac{1}{3} = \log_{2^{-3}} \sqrt{3x-5}$

$\frac{1}{6}\log_2(x-2) + \frac{1}{3}\log_2 \sqrt{3x-5} = \frac{1}{3}$

$\frac{1}{6}\log_2(x-2) + \frac{1}{6}\log_2(3x-5) = \frac{1}{3} \quad \Big| \cdot 6$

$\log_2(x-2) + \log_2(3x-5) = 2$

$\log_2(x-2)(3x-5) = 2$

$\log_2(3x^2 - 5x - 6x + 10) = 2$

$\log_2(3x^2 - 11x + 10) = 2$

$3x^2 - 11x + 10 = 2^2 = 4$

$3x^2 - 11x + 6 = 0$

$x_{1,2} = \frac{11 \pm \sqrt{121 - 72}}{6} \rightarrow x_1 = 3, \quad \cancel{x_2 = \frac{2}{3}}$

Rešenje: $x = 3$

e) $\log_x 2 - \log_4 x + \frac{7}{6} = 0, \ x \neq 1$

$\log_x 2 - \log_{2^2} x + \frac{7}{6} = 0$

$\log_x 2 - \frac{1}{2}\log_2 x + \frac{7}{6} = 0$

$\frac{1}{\log_2 x} - \frac{1}{2}\log_2 x + \frac{7}{6} = 0 \quad | \cdot \log_2 x$

$1 - \frac{1}{2}(\log_2 x)^2 + \frac{7}{6}\log_2 x = 0 \quad | \cdot 6$

$6 - 3(\log_2 x)^2 + 7\log_2 x = 0 \qquad t = \log_2 x$

$6 - 3t^2 + 7t = 0$

$t_{1,2} = \frac{-7 \pm \sqrt{49+72}}{-6} \nearrow x_1 = -\frac{2}{3}$

$\searrow x_2 = 3$

$\log_2 x = -\frac{2}{3}$

$x = 2^{-\frac{2}{3}} = \frac{1}{2^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{4}}$

$\log_2 x = 3$

$x = 2^3 = 8$

8 a) $\log_5(3x-1) < 1$

$3x - 1 > 0$

$3x > 1$

$x > \frac{1}{3}$

$\log_5(3x-1) < \log_5 5$

$3x - 1 < 5$

$3x < 6$

$x < 2$

$x \in \left(\frac{1}{3}, 2\right)$

b) $\log_{\frac{1}{5}}(5x-1) > 0$

$5x - 1 > 0$

$5x > 1$

$x > \frac{1}{5}$

$\log_{\frac{1}{5}}(5x-1) > \log_{\frac{1}{5}} 1$

$5x - 1 < 1$

$5x < 2$

$x < \frac{2}{5}$

$x \in \left(\frac{1}{5}, \frac{2}{5}\right)$

c) $\log_{0,5}(x^2-5x+6) > -1$

$x^2-5x+6>0$

$x_{1,2}=\frac{5\pm\sqrt{25-24}}{2}$ $\nearrow x_1=3$, $\searrow x_2=2$

$x\in(-\infty,2)\cup(3,+\infty)$

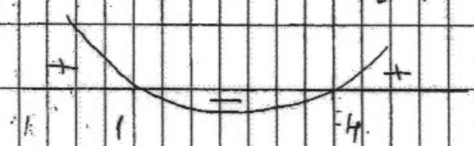
$\log_{0,5}(x^2-5x+6) > \log_{0,5}\left(\frac{1}{2}\right)^{-1}$

$x^2-5x+6<\left(\frac{1}{2}\right)^{-1}$

$x^2-5x+6<2$

$x^2-5x+4<0$

$x_{1,2}=\frac{5\pm\sqrt{25-16}}{2}$ $\nearrow x_1=4$, $\searrow x_2=1$



$x\in(1,4)$

Rešenje: $(1,2)\cup(3,4)$

d) $\log_3\frac{1+2x}{1+x}<1$

$\frac{1+2x}{1+x}>0$

$1+2x=0$ $\quad 1+x=0$

$2x=-1$ $\quad x=-1$

$x=-\frac{1}{2}$

	$-\infty$	-1		$-\frac{1}{2}$	$+\infty$
$1+2x$	$-$		$-$	0	$+$
$1+x$	$-$	0	$+$		$+$
$\frac{1+2x}{1+x}$	$+$		$-$		$+$

$x\in(-\infty,-1)\cup\left(-\frac{1}{2},+\infty\right)$

$\log_3\frac{1+2x}{1+x}<\log_3 3$

$\frac{1+2x}{1+x}<3$

$\frac{1+2x-3-3x}{1+x}<0$

$\frac{-x-2}{x+1}<0$

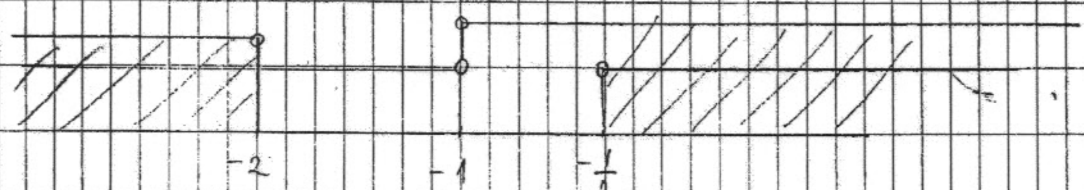
$-x-2=0$

$-x=2$

$x=-2$

	$-\infty$	-2		-1	$+\infty$
$-x-2$	$+$	0	$-$		$-$
$x+1$	$-$		$-$	0	$+$
	$-$		$+$		$-$

$x\in(-\infty,-2)\cup(-1,+\infty)$



$x\in(-\infty,-2)\cup\left(-\frac{1}{2},+\infty\right)$

9. a) $\log_3 x + \log_3 y = 2 + \log_3 2$ za $x > 0,\ y > 0$

$\log_{27}(x+y) = \frac{2}{3}$

$\log_3(xy) - \log_3 2 = 2$

$\log_3\left(\frac{xy}{2}\right) = 2$

$\log_{3^3}(x+y) = \frac{2}{3}$

$\frac{1}{3}\log_3(x+y) = \frac{2}{3}$

$\log_3(x+y) = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$

$\frac{xy}{2} = 3^2 = 9$

$xy = 18$

$x + y = 3^2 = 9$

$y = 9 - x$

$x(9-x) = 18$

$9x - x^2 - 18 = 0$

$-x^2 + 9x - 18 = 0 \quad | \cdot (-1)$

$x^2 - 9x + 18 = 0$

$x_{1,2} = \frac{9 \pm \sqrt{81-72}}{2}$ → $x_1 = 6$, $x_2 = 3$

$x_1 = 6,\ y_1 = 3 \quad (x,y) = (6,3)$

$x_2 = 3,\ y_2 = 6 \quad (x,y) = (3,6)$

b) $2(\log_y x + \log_x y) = 5$ za $x > 0,\ y > 0,\ x \neq 1,\ y \neq 1$

$xy = 8$

$2\log_y x + \frac{2}{\log_y x} = 5 \quad t = \log_y x$

$2t + \frac{2}{t} = 5 \quad | \cdot t$

$2t^2 - 5t + 2 = 0$

$t_{1,2} = \frac{5 \pm \sqrt{25-16}}{4}$ → $t_1 = 2$, $t_2 = \frac{1}{2}$

$\log_y x = 2$

$y^2 = x$

$xy = 8$

$y^3 = 8$

$y = 2$

$x = 4$

$\log_y x = \frac{1}{2}$

$y^{\frac{1}{2}} = x$

$y = x^2$

$xy = 2$

$x^3 = 8$

$x = 2$

$y = 4$

3.3. Zadaci za vežbu

1. $4^x - 10 \cdot 2^{x-1} = 24$

$2^{2x} - 5 \cdot 2^1 \cdot 2^{x-1} = 24$

$2^{2x} - 5 \cdot 2^x = 24 \qquad 2^x = t$

$t^2 - 5t - 24 = 0$

$t_{1,2} = \dfrac{5 \pm \sqrt{25+96}}{2} \quad \begin{matrix} \nearrow t_1 = 8 \\ \searrow \cancel{t_2 = -3} \end{matrix}$

$2^x = 8$

$\boxed{x = 3}$

2. $64 \cdot 9^x - 84 \cdot 12^x + 27 \cdot 16^x = 0$

$4^3 \cdot 3^{2x} - 84 \cdot (3 \cdot 4)^x + 3^3 \cdot 4^{2x} = 0 \quad | : 3^x 4^x$

$4^3 \cdot \dfrac{3^x}{4^x} - 84 + 3^3 \cdot \dfrac{4^x}{3^x} = 0 \qquad t = \dfrac{3^x}{4^x}$

$64 \cdot t - 84 + 27 \cdot \dfrac{1}{t} = 0 \quad | \cdot t$

$64t^2 - 84t + 27 = 0$

$t_{1,2} = \dfrac{84 \pm \sqrt{7056 - 6912}}{128} \quad \begin{matrix} \nearrow t_1 = \frac{3}{4} \\ \searrow t_2 = \frac{9}{16} \end{matrix}$

$\dfrac{3^x}{4^x} = \dfrac{3}{4} \qquad \dfrac{3^x}{4^x} = \dfrac{9}{16}$

$\boxed{x = 1} \qquad \boxed{x = 2}$

3. $4^{-x+\frac{1}{2}} - 7 \cdot 2^{-x} - 4 < 0$

$4^{-x} \cdot 4^{\frac{1}{2}} - 7 \cdot 2^{-x} - 4 < 0$

$2^{-2x} \cdot 2 - 7 \cdot 2^{-x} - 4 < 0 \qquad 2^{-x} = t$

$2t^2 - 7t - 4 < 0$

$t_{1,2} = \dfrac{7 \pm \sqrt{49+32}}{4} \quad \begin{matrix} \nearrow t_1 = 4 \\ \searrow \cancel{t_2 = -\frac{1}{2}} \end{matrix}$

$2^{-x} = 4$

$2^{-x} = 2^2$

$-x = 2$

$x = -2$

$2^{-x} < 4$

$2^{-x} < 2^2$

$-x < 2$

$\boxed{x > -2}$

4. $3^{2x} - 2^{y} = 77$

$3^{x} - 2^{\frac{y}{2}} = 7$

$3^{x} = a$

$2^{\frac{y}{2}} = b$

$a^2 - b^2 = 77$

$a - b = 7$

$(a-b)(a+b) = 77$

$7(a+b) = 77$

$a + b = 11$

$a - b = 7$

$2a = 18$

$a = 9$

$b = 2$

$3^{x} = 9$

$3^{x} = 3^{2}$

$\boxed{x = 2}$

$2^{\frac{y}{2}} = 2^{1}$

$\frac{y}{2} = 1$

$\boxed{y = 2}$

5. $\left(\log_{\frac{1}{2}}(4x)\right)^2 + \log_2 \frac{x^2}{8} = 8$

$\left(\log_{\frac{1}{2}}(4x)\right)^2 + \log_2 x^2 - \log_2 8 = 8$

$\left(\log_{\frac{1}{2}} 4 + \log_{\frac{1}{2}} x\right)^2 + 2\log_2 x - 3 = 8$

$\left(\log_{\frac{1}{2}} x - 2\right)^2 + 2\log_{(\frac{1}{2})^{-1}} x = 11$

$\left(\log_{\frac{1}{2}} x\right)^2 - 4\log_{\frac{1}{2}} x + 4 - 2\log_{\frac{1}{2}} x = 11$

$\left(\log_{\frac{1}{2}} x\right)^2 - 6\log_{\frac{1}{2}} x - 7 = 0$ $\qquad \log_{\frac{1}{2}} x = t$

$t^2 - 6t - 7 = 0$

$t_{1,2} = \frac{6 \pm \sqrt{36 + 28}}{2}$ $\to t_1 = 7$, $\to t_2 = -1$

$\log_{\frac{1}{2}} x = 7$

$x = \left(\frac{1}{2}\right)^7$

$\boxed{x = \frac{1}{128}}$

$\log_{\frac{1}{2}} x = -1$

$x = \left(\frac{1}{2}\right)^{-1}$

$\boxed{x = 2}$

16.

$$\log_x 3 + \log_3 x = \log_{\sqrt{x}} 3 + \log_3 \sqrt{x} + \frac{1}{2}$$

$$\log_x 3 + \log_3 x = 2\log_x 3 + \frac{1}{2}\log_3 x + \frac{1}{2}$$

$$\log_x 3 + \frac{1}{\log_x 3} - 2\log_x 3 - \frac{1}{2\log_x 3} - \frac{1}{2} = 0 \quad \Big| \cdot 2\log_x 3$$

$$2(\log_x 3)^2 + 2 - 4(\log_x 3)^2 - 1 - \log_x 3 = 0$$

$$-2(\log_x 3)^2 - \log_x 3 + 1 = 0 \qquad \log_x 3 = t$$

$$-2t^2 - t + 1 = 0$$

$$t_{1,2} = \frac{1 \pm \overset{3}{\sqrt{1+8}}}{-4} \quad t_1 = -1, \quad t_2 = \frac{1}{2}$$

$\log_x 3 = -1$

$x^{-1} = 3$

$\frac{1}{x} = 3$

$\boxed{x = \frac{1}{3}}$

$\log_x 3 = \frac{1}{2}$

$x^{\frac{1}{2}} = 3$

$\sqrt{x} = 3$

$\boxed{x = 9}$

7.

$$\log_{\frac{1}{4}} (2-x) > \log_{\frac{1}{4}} \frac{2}{x+1}$$

$2 - x > 0$ $\qquad \frac{2}{x+1} > 0$

$-x > -2$ $\qquad x + 1 > 0$

$x < 2$ $\qquad x > -1$

$$2 - x < \frac{2}{x+1}$$

$$2 - x - \frac{2}{x+1} < 0$$

$$\frac{(2-x)(x+1) - 2}{x+1} < 0$$

$$\frac{2x + \cancel{2} - x^2 - x - \cancel{2}}{x+1} < 0$$

$$\frac{-x^2 + x}{x+1} < 0$$

$$\frac{x(1-x)}{x+1} < 0$$

	$-\infty$		-1		0		1		$+\infty$
x		$-$		$-$	0	$+$		$+$	
$1-x$		$+$		$+$		$+$	0	$-$	
$x+1$		$-$	0	$+$		$+$		$+$	
		$+$		$-$		$+$		$-$	

$x \in (-1, 0) \cup (1, +\infty)$

Uz uslov: $x \in (-1, 0) \cup (1, 2)$

8. $\log_2 x + \log_4 y = -2\log_{\frac{1}{2}} 4$

$\log_4 x + \log_2 y = 5$

$\log_2 x + \log_{2^2} y = 4$

$\log_{2^2} x + \log_2 y = 5$

$\log_2 x + \frac{1}{2}\log_2 y = 4 \quad | \cdot (-2)$

$\frac{1}{2}\log_2 x + \log_2 y = 5$

$-2\log_2 x - \log_2 y = -8$

$\frac{1}{2}\log_2 x + \log_2 y = 5$

$-\frac{3}{2}\log_2 x = -3$

$\log_2 x = \frac{-\cancel{3}\frac{1}{}}{-\frac{\cancel{3}}{2}}$

$\log_2 x = 2$

$x = 2^2$

$\boxed{x = 4}$

$2 + \frac{1}{2}\log_2 y = 4$

$\frac{1}{2}\log_2 y = 2$

$\log_2 y = 4$

$y = 2^4$

$\boxed{y = 16}$

9. $2^{3x-2} - 2^{3x-3} - 2^{3x-4} - 4 = 0$

$2^{3x} \cdot 2^{-2} - 2^{3x} \cdot 2^{-3} - 2^{3x} \cdot 2^{-4} = 2^2$

$2^{3x}(2^{-2} - 2^{-3} - 2^{-4}) = 2^2$

$2^{3x} \cdot \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{16}\right) = 2^2$

$2^{3x}\left(\frac{4}{16} - \frac{2}{16} - \frac{1}{16}\right) = 2^2$

$2^{3x} = \frac{2^2}{\frac{1}{2^4}}$

$2^{3x} = 2^6$

$3x = 6$

$\boxed{x = 2}$

10. $\left(\frac{9}{25}\right)^{4x^2+4x-11} \cdot \left(\frac{5}{3}\right)^{2x+1} = \left(\frac{5}{3}\right)^9$

$\left(\frac{5}{3}\right)^{-2(4x^2+4x-11)} \cdot \left(\frac{5}{3}\right)^{2x+1} = \left(\frac{5}{3}\right)^9$

$-8x^2 - 8x + 22 + 2x + 1 = 9$

$-8x^2 - 6x + 14 = 0 \quad | : (-2)$

$4x^2 + 3x - 7 = 0$

$x_{1,2} = \frac{-3 \pm \sqrt{9 + 112}}{8} \rightarrow x_1 = 1, \quad x_2 = -\frac{7}{4}$

11. $3^{\frac{2}{x}} - 12 \cdot 3^{\frac{1}{x}} + 27 = 0 \qquad t = 3^{\frac{1}{x}}$

$t^2 - 12t + 27 = 0$

$t_{1,2} = \frac{12 \pm \sqrt{144 - 108}}{2} \begin{cases} t_1 = 9 \\ t_2 = 3 \end{cases}$

$3^{\frac{1}{x}} = 3^2 \qquad 3^{\frac{1}{x}} = 3^1$

$\frac{1}{x} = 2 \qquad \frac{1}{x} = 1$

$\boxed{x = \frac{1}{2}} \qquad \boxed{x = 1}$

12. $5^{\frac{x+1}{1-2x}} < 0{,}2^{-3}$

$5^{\frac{x+1}{1-2x}} < 5^3$

$\frac{x+1}{1-2x} < 3$

$\frac{x+1-3+6x}{1-2x} < 0$

$\frac{7x-2}{1-2x} < 0$

$7x - 2 = 0$

$7x = 2$

$x = \frac{2}{7}$

$1 - 2x = 0$

$-2x = -1$

$x = \frac{1}{2}$

	$-\infty$	$\frac{2}{7}$	$\frac{1}{2}$ $+\infty$
$7x-2$	$-$ 0	$+$	$+$
$1-2x$	$+$	$+$ 0	$-$
	$-$	$+$	$-$

$x \in \left(-\infty, \frac{2}{7}\right) \cup \left(\frac{1}{2}, +\infty\right)$

13. $\log_{10} \sqrt{75 + 5^{\sqrt[3]{x-1}}} = 1$

$\frac{1}{2} \log_{10} \left(75 + 5^{\sqrt[3]{x-1}}\right) = 1$

$\frac{1}{2} \log_{10} \left(75 + 5^{\sqrt[3]{x-1}}\right) = \frac{1}{2} \log_{10} 100$

$75 + 5^{\sqrt[3]{x-1}} = 100$

$5^{\sqrt[3]{x-1}} = 100 - 75$

$5^{\sqrt[3]{x-1}} = 25$

$5^{\sqrt[3]{x-1}} = 5^2$

$\sqrt[3]{x-1} = 2$

$x - 1 = 2^3$

$x - 1 = 8$

$\boxed{x = 9}$

14. $3\sqrt{\ln x} + \ln x = 4$ $\quad t = \sqrt{\ln x}$

$3t + t^2 - 4 = 0$

$t_{1,2} = \frac{-3 \pm \sqrt{9+16}}{2} \to t_1 = 1$, $\cancel{t_2 = -4}$

$\sqrt{\ln x} = 1$

$\ln x = 1$

$\log_e x = 1$

$\boxed{x = e}$

15. $\log_{10} x + \frac{4}{3}\log_{10} 5 - 1 = 3\cdot(1 - \log_{10} 5)$

$\log_{10} x + \log_{10}\sqrt[3]{5^4} - \log_{10} 10 = 3\log_{10} 10 - 3\log_{10} 5$

$\log_{10}\left(\frac{\sqrt[3]{5^4}x}{10}\right) = 3\log_{10}\left(\frac{10}{5}\right)$

$\log_{10}\left(\frac{\sqrt[3]{5^4}x}{10}\right) = \log_{10} 8$

$\frac{\sqrt[3]{5^4}x}{10} = 8$

$\sqrt[3]{5^4}x = 80$

$x = \frac{80}{\sqrt[3]{5^4}} = \frac{80}{\sqrt[3]{5^3}\cdot\sqrt[3]{5}} = \frac{16}{\sqrt[3]{5}}\cdot\frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$

$x = \frac{16\sqrt[3]{25}}{5}$

16. $\log_3 x - \log_9 x + \log_{81} x = \frac{3}{4}$

$\log_3 x - \frac{1}{2}\log_3 x + \frac{1}{4}\log_3 x = \frac{3}{4} \quad |\cdot 4$

$4\log_3 x - 2\log_3 x + \log_3 x = 3$

$3\log_3 x = 3$

$\log_3 x = 1$

$x = 3^1$

$x = 3$

17. $\ln\frac{2x+1}{2} + \ln x + \ln 2 = 0$

$\ln(2x+1) - \cancel{\ln 2} + \ln x + \cancel{\ln 2} = 0$

$\ln(2x+1) = \ln x^{-1}$

$2x+1 = \frac{1}{x} \quad | \cdot x$

$2x^2 + x = 1$

$2x^2 + x - 1 = 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{4} \rightarrow \boxed{x_1 = \frac{1}{2}}$, $\cancel{x_2 = -1}$

$\frac{2x+1}{2} > 0 \qquad x > 0$

$2x+1 > 0$

$2x > -1$

$x > -\frac{1}{2}$

18. $\log_9 x + \log_3 2 = \log_{\frac{1}{3}} \sqrt{3}$

$\log_{3^2} x + \log_3 2 = \log_{3^{-1}} \sqrt{3}$

$\frac{1}{2}\log_3 x + \log_3 2 = -\frac{1}{2}\log_3 3 \quad | \cdot 2$

$\log_3 x + 2\log_3 2 = -\log_3 3$

$\log_3 x + \log_3 4 = \log_3 \frac{1}{3}$

$\log_3 (x4) = \log_3 \frac{1}{3}$

$4x = \frac{1}{3}$

$x = \frac{\frac{1}{3}}{\frac{4}{1}}$

$\boxed{x = \frac{1}{12}}$

19. $\log_{x-2}(x^2 - 6x + 10) - 1 = 0$

$\log_{x-2}(x^2 - 6x + 10) = \log_{x-2}(x-2)$

$x^2 - 6x + 10 = x - 2$

$x^2 - 7x + 12 = 0$

$x_{1,2} = \frac{7 \pm \sqrt{49-48}}{2} \rightarrow x_1 = 4$, $\cancel{x_2 = 3}$

$x - 2 \neq 1$

$x \neq 3$

20. $2\log_{x+1}5 = \log_5(x+1) - 1$

$\frac{2}{\log_5(x+1)} - \log_5(x+1) + 1 = 0 \quad | \cdot \log_5(x+1)$

$2 - (\log_5(x+1))^2 + \log_5(x+1) = 0 \qquad \log_5(x+1) = t$

$-t^2 + t + 2 = 0$

$t_{1,2} = \frac{-1 \pm \sqrt{1+8}}{-2}$ $\to t_1 = -1$, $\to t_2 = 2$

$\log_5(x+1) = -1$

$x+1 = 5^{-1}$

$x + 1 = \frac{1}{5}$

$x = \frac{1}{5} - \frac{5}{5}$

$\boxed{x = -\frac{4}{5}}$

$\log_5(x+1) = 2$

$x + 1 = 5^2$

$x + 1 = 25$

$\boxed{x = 24}$

21. $x^{2\log_x 10} - 10x = 0$

$10^2 - 10x = 0$

$100 - 10x = 0$

$10x = 100$

$\boxed{x = 10}$

22. $x^2 - 4x - \log_2 m = 0$

$D \geq 0$

$16 + 4\log_2 m \geq 0 \quad | : 4$

$\log_2 m \geq -4$

$\log_2 m \geq \log_2 \frac{1}{16}$

$m \geq \frac{1}{6}$

$m \in [\frac{1}{16}, +\infty)$

23. $\ln\left|\frac{x-1}{2x+1}\right| < 0$

$\left|\frac{x-1}{2x+1}\right| < 1$

$\left|\frac{x-1}{2x+1}\right| = \begin{cases} \frac{x-1}{2x+1}, & x \in (-\infty, -\frac{1}{2}) \cup [1, +\infty) \\ -\frac{x-1}{2x+1}, & x \in (-\frac{1}{2}, 1) \end{cases}$

	$-\infty$	$-\frac{1}{2}$		1		$+\infty$
$x-1$	$-$		$-$	0	$+$	
$2x+1$	$-$	0	$+$		$+$	
	$+$		$-$		$+$	

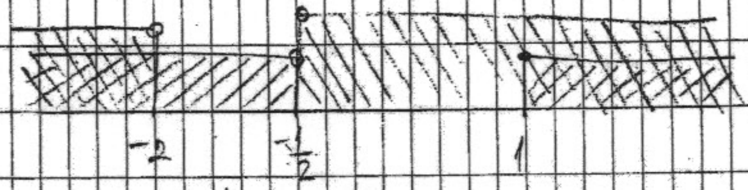
$\frac{x-1}{2x+1} < 1$

$\frac{x-1-2x-1}{2x+1} < 0$

$\frac{-x-2}{2x+1} < 0$

	$-\infty$	-2		$-\frac{1}{2}$		$+\infty$
$-x-2$	$+$	0	$-$		$-$	
$2x+1$	$-$		$-$	0	$+$	
	$-$		$+$		$-$	

$x \in (-\infty, -2) \cup (-\frac{1}{2}, +\infty)$



$R_1: x \in (-\infty, -2) \cup (1, +\infty)$

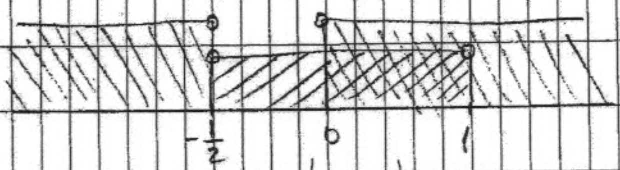
$\frac{-x+1}{2x+1} < 1$

$\frac{-x+1-2x-1}{2x+1} < 0$

$\frac{-3x}{2x+1} < 0$

	$-\infty$	$-\frac{1}{2}$		0		$+\infty$
$-3x$	$+$		$+$	0	$-$	
$2x+1$	$-$	0	$+$		$+$	
	$-$		$+$		$-$	

$x \in (-\infty, -\frac{1}{2}) \cup (0, +\infty)$



$R_2: x \in (0, 1)$

$R: x \in (-\infty, -2) \cup (0, 1) \cup (1, +\infty)$

24. $\sqrt{\log_2 \frac{2x-3}{x-1}} < 1$

$\log_2 \frac{2x-3}{x-1} \geq 0$

$\frac{2x-3}{x-1} \geq 1$

$\frac{2x-3-x+1}{x-1} \geq 0$

$\frac{x-2}{x-1} \geq 0$

	$-\infty$	1		2		$+\infty$
$x-2$	$-$		$-$	0	$+$	
$x-1$	$-$	0	$+$		$+$	
	$+$		$-$		$+$	

$x \in (-\infty, 1) \cup (2, +\infty)$

$\log_2 \frac{2x-3}{x-1} < \log_2 2$

$\frac{2x-3}{x-1} < 2$

$\frac{\cancel{2x}-3-\cancel{2x}+2}{x-1} < 0$

$\frac{-1}{x-1} < 0$

$x-1 > 0$

$x > 1$

$R: x \in (2, +\infty)$

24.

$$3^{3x-2y} = 27$$
$$2^{x+y-1} = 32$$

$$3^{3x-2y} = 3^3$$
$$2^{x+y-1} = 2^5$$
$$x+y-1=5$$
$$x+y=6$$
$$y=6-x$$

$$3x-2y=3$$
$$3x-2(6-x)=3$$
$$3x-12+2x=3$$
$$5x=15$$
$$\boxed{x=3 \quad y=3}$$

26.

$$\log_2 x - \log_4 y = 0$$
$$5x^2 - y^2 = 4$$
$$\log_2 x - \frac{1}{2}\log_2 y = 0$$
$$\log_2 x = \log_2 \sqrt{y}$$
$$x = \sqrt{y}$$
$$x^2 = y$$

$$5x^2 - x^4 = 4 \qquad t = x^2$$
$$5t - t^2 - 4 = 0$$
$$t_{1,2} = \frac{-5 \pm \sqrt{25-16}}{-2} \quad \to t_1 = 4, \quad \to t_2 = 1$$
$$x_1^2 = 4$$
$$\boxed{x_1 = 2, \; y_1 = 4}$$
$$x_2^2 = 1$$
$$\boxed{x_2 = 1, \; y_2 = 1}$$

27.

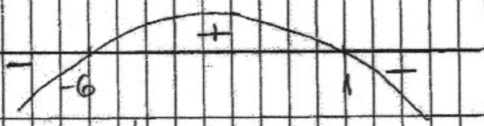
$$3^{x+2} + 9^{x+1} = 810$$
$$3^{x+2} + 3^{2x+2} = 10 \cdot 3^4$$
$$10 = 3^{x-2} + 3^{2x-2}$$
$$\frac{3^x + 3^{2x}}{3^2} = 10$$
$$3^x + 3^{2x} = 90$$
$$3^{2x} + 3^x - 90 = 0 \qquad t = 3^x$$
$$t^2 + t - 90 = 0$$
$$t_{1,2} = \frac{-1 \pm \sqrt{1+360}}{2} \quad \to t_1 = 9, \quad \to \cancel{t_2 = -10}$$
$$3^x = 9 = 3^2$$
$$\boxed{x = 2}$$

28. $\log_x(6-5x)<2$ $\quad$ $\log_x(6-5x)<\log_x x^2$

$6-5x>0$ $\quad$ $6-5x<x^2$

$-5x>-6$ $\quad$ $-x^2-5x+6<0$

$x<\frac{6}{5}$ $\quad$ $x_{1,2}=\frac{5\pm\sqrt{25+24}}{-2}$ → $x_1=-6$, $x_2=1$



$x\in\left(1,\frac{6}{5}\right)$

31. $3^{2x}-2^y=77$ $\quad$ $3^x=a$ $\quad$ $b=2$ $\quad$ $2^{\frac{y}{2}}=2^1$

$3^x-2^{\frac{y}{2}}=7$ $\quad$ $2^{\frac{y}{2}}=b$ $\quad$ $3^x=9$ $\quad$ $\frac{y}{2}=1$

$a^2-b^2=77$ $\quad$ $3^x=3^2$ $\quad$ $\boxed{y=2}$

$a-b=7$ $\quad$ $\boxed{x=2}$

$(a-b)(a+b)=77$ $\quad$ $(x,y)=(2,2)$

$7a+7b=77$

$a+b=11$

$a-b=7$

$2a=18$

$a=9$

39. $\log_2(x+1)=\log_4(x+3)$ $\quad$ $\log_2(x+1)=\log_2\left(\sqrt{x+3}\right)$

$x+1>0$ $\quad$ $x+1=\sqrt{x+3}$ $\;|^2$

$\boxed{x>-1}$ $\quad$ $x^2+2x+1=x+3$

$x+3>0$ $\quad$ $x^2+x-2=0$

$x>-3$ $\quad$ $x_{1,2}=\frac{-1\pm\sqrt{1+8}}{2}$ → $x_1=1$, $\cancel{x_2=-2}$

$\boxed{x=1}$

40. $2^x \cdot 3^{y-2} = 4$

$2^x + \sqrt{3^{2y}} = 13$

$2^x \cdot 3^y \cdot 3^{-2} = 4$

$2^x \cdot 3^y = \frac{4}{\frac{1}{9}} = 36$ $\qquad 2^x = a$

$2^x + 3^y = 13$ $\qquad 3^y = b$

$a \cdot b = 36$

$a + b = 13$

$a = 13 - b$

$(13-b) \cdot b = 36$

$13b - b^2 - 36 = 0$

$b_{1,2} = \frac{-13 \pm \sqrt{169-144}}{-2} \to b_1 = 4 \quad a_1 = 9$

$\to b_2 = 9 \quad a_2 = 4$

$3^{y_1} = 4$

$\boxed{y_1 = \log_3 4}$

$2^{x_1} = 9$

$\boxed{x_1 = \log_2 9}$

$3^{y_2} = 9$

$\boxed{y_2 = 2}$

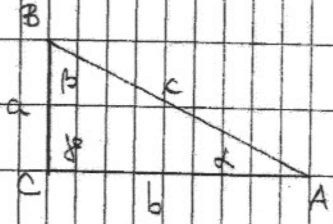
$2^{x_2} = 4$

$\boxed{x_2 = 2}$

4. Trigonometrija

4.1. Osnovne trigonometrijske funkcije

$180° = \pi$



$\sin\alpha = \frac{a}{c} \qquad \cos\alpha = \frac{b}{c} \qquad tg\alpha = \frac{a}{b} \qquad ctg\alpha = \frac{b}{a}$

$\sin\beta = \frac{b}{c} \qquad \cos\beta = \frac{a}{c} \qquad tg\beta = \frac{b}{a} \qquad ctg\beta = \frac{a}{b}$

$\sin\alpha = \cos\beta \qquad \sin\beta = \cos\alpha \qquad tg\alpha = ctg\beta \qquad tg\beta = ctg\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \qquad \left(\frac{a+b}{c}\right)^2 = \frac{(a+b)^2}{c^2} = \frac{c^2}{c^2} = 1$

$-1 \le \sin\alpha \le 1 \qquad -1 \le \cos\beta \le 1 \qquad tg(\pi - \alpha) = -tg\alpha \qquad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$ 3

$\sin(\pi - \alpha) = \sin\alpha \qquad \sin(\pi + \alpha) = -\sin\alpha \qquad tg(-\alpha) = -tg\alpha \qquad tg\left(\frac{\pi}{2} - \alpha\right) = ctg\alpha$

$\sin(-\alpha) = -\sin\alpha \qquad \sin(2\pi - \alpha) = -\sin\alpha \qquad ctg(\pi - \alpha) = -ctg\alpha \qquad ctg\left(\frac{\pi}{2} - \alpha\right) = tg\alpha$

$\cos(\pi - \alpha) = -\cos\alpha \qquad \cos(\pi + \alpha) = -\cos\alpha \qquad ctg(-\alpha) = -ctg\alpha$

$\cos(-\alpha) = \cos\alpha \qquad \cos(2\pi - \alpha) = \cos\alpha \qquad \sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$