2. Linearne i kvadratne jednačine i nejednačine

2.1. LINEARNA JEDNAČINA I NEJEDNAČINA

$ax + b = 0$ - opšti oblik linearne jednačine $\Rightarrow x = -\frac{b}{a}$

$y = ax + b$ - jednačina prave
a - koeficijent pravca
b - odsečak na y osi

Za $a \neq 0$ $x = -\frac{b}{a}$ - apscisa presečne tačke prave i x-ose.
Za $a > 0$ $y \nearrow$, pa je $y > 0$ za $x > -\frac{b}{a}$ i $y < 0$ za $x < -\frac{b}{a}$
Za $a < 0$ $y \searrow$, pa je $y > 0$ za $x < -\frac{b}{a}$ i $y < 0$ za $x > -\frac{b}{a}$

* Specijalni slučajevi

Za $a = 0$, prava $y = b$ je paralelna x-osi
Prava $x = c$, $c \in R$ je paralelna y-osi.

1.

a) $|3x - 2| + x = 11$

$3x - 2 \geq 0$ ili $3x - 2 < 0$

$3x \geq 2$ | $3x < 2$

$\boxed{x \geq \frac{2}{3}}$ | $\boxed{x < \frac{2}{3}}$

$3x - 2 + x = 11$
$4x = 13$
$x = \frac{13}{4}$

$-3x + 2 + x = 11$
$-2x = 9$
$x = -\frac{9}{2}$

b) $|x+2| = 2(3-x)$

$x+2 \geq 0$ ili $x+2 < 0$

$x \geq -2$ $\quad$ $x < -2$

$x+2 = 6-2x$

$3x = 4$

$x = \frac{4}{3}$ ✓

$-x-2 = 6-2x$

$x = 8$

Nije rešenje.

2.2. KVADRATNA JEDNAČINA I NEJEDNAČINA

$ax^2+bx+c=0$, $a \neq 0$ - opšti oblik kvadratne jednačine

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$ax^2+bx+c = a(x-x_1)(x-x_2)$$

$D = b^2-4ac$ - diskriminanta kvadratne jednačine

$D>0 \Rightarrow x_1, x_2 \in R$ i $x_1 \neq x_2$ (dva različita realna rešenja)

$D=0 \Rightarrow x_1, x_2 \in R$ i $x_1 = x_2$ (jedno realno rešenje)

$D<0 \Rightarrow$ jednačina nema realnih rešenja - rešenja su kompleksni brojevi: $x_{1,2} = -\frac{b}{2a} \pm i\frac{\sqrt{4ac-b^2}}{2a}$

$y = ax^2+bx+c$, $a \neq 0$ je kvadratna funkcija

Ako je $D>0$, grafik funkcije seče osu u tačkama $T_1(x_1,0)$ i $T_2(x_2,0)$ gde su x_1 i x_2 nule funkcije.

Ako je $D=0$, grafik funkcije dodiruje osu u tački $T(x_1,0)$.

Ako je $D<0$, grafik funkcije ne seče x-osu.

Za $a>0$, grafik kvadratne funkcije je parabola sa otvorom nagore

Za $a<0$, grafik funkcije je parabola sa otvorom nadole.

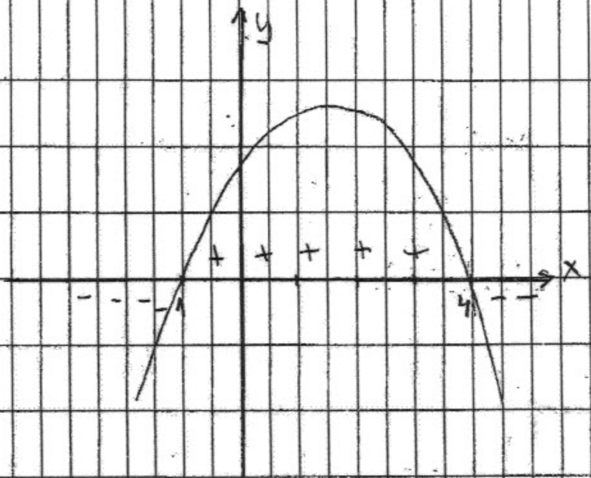
Teme parabole je tačka $T\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

2.

a) $y = 4 + 3x - x^2$

$y = 0 \Rightarrow -x^2 + 3x + 4 = 0$

$x_{1,2} = \frac{-3 \pm \overset{5}{\sqrt{9+16}}}{-2} \rightarrow x_1 = -1, \quad x_2 = 4$



$y = 0$ za $x \in \{-1, 4\}$

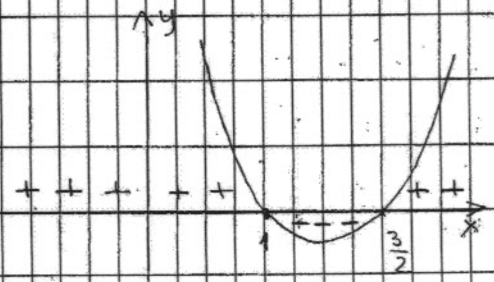
$y > 0$ za $x \in (-1, 4)$

$y < 0$ za $x \in (-\infty, -1) \cup (4, +\infty)$

b) $y = 2x^2 - 5x + 3$

$y = 0 \Rightarrow 2x^2 - 5x + 3 = 0$

$x_{1,2} = \frac{5 \pm \overset{1}{\sqrt{25-24}}}{4} \rightarrow x_1 = \frac{3}{2}, \quad x_2 = 1$



$y = 0$ za $x \in \{1, \frac{3}{2}\}$

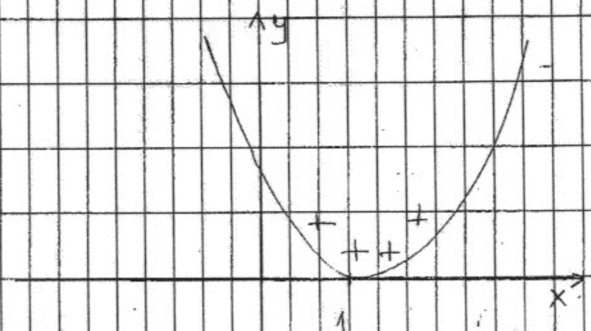
$y > 0$ za $x \in (-\infty, 1) \cup (\frac{3}{2}, +\infty)$

$y < 0$ za $x \in (1, \frac{3}{2})$

c) $y = x^2 - 2x + 1$

$y = 0 \Rightarrow x^2 - 2x + 1 = 0$

$x_{1,2} = \frac{2 \pm \overset{0}{\sqrt{4-4}}}{2} \rightarrow x_1 = 1$



$y = 0$ za $x = 1$

$y > 0$ za $x \in (-\infty, 1) \cup (1, +\infty)$

d) $y = x^2 + 4x + 5$

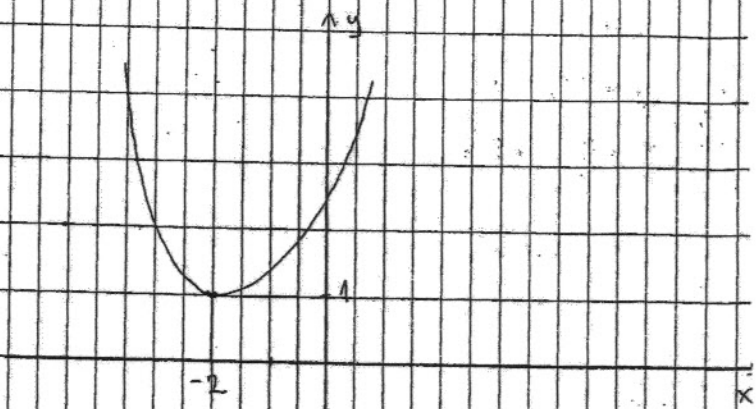
$y = 0 \Rightarrow x^2 + 4x + 5 = 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16-20}}{2}$

$D = -4$

$y > 0$ za svako $x \in R$

$T\left(-\frac{4}{2}, \frac{20-16}{4}\right)$ $T(-2, 1)$



e) $y = 3x^2 + 2x$

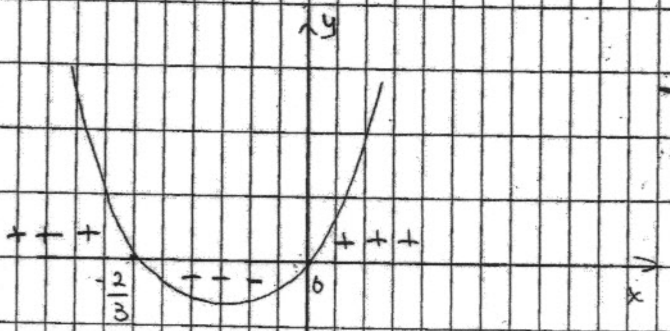
$y = 0 \Rightarrow 3x^2 + 2x = 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4-0}}{6}$, $x_1 = 0$, $x_2 = -\frac{2}{3}$

$y = 0$ za $x \in \left\{-\frac{2}{3}, 0\right\}$

$y > 0$ za $x \in \left(-\infty, -\frac{2}{3}\right) \cup (0, +\infty)$

$y < 0$ za $x \in \left(-\frac{2}{3}, 0\right)$



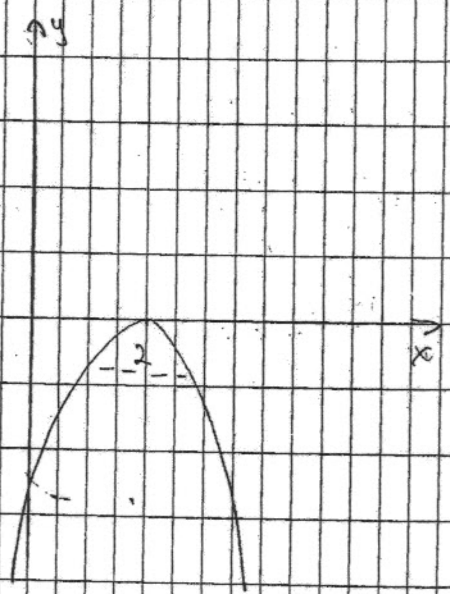
f) $y = -x^2 + 4x - 4$

$y = 0 \Rightarrow -x^2 + 4x - 4 = 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16-16}}{-2} \Rightarrow x_1 = 2$

$y = 0$ za $x = 2$

$y < 0$ za $x \in (-\infty, 2) \cup (2, +\infty)$



g) $y = -\frac{1}{2}x^2 + x - 2$

$y = 0 \Rightarrow -\frac{1}{2}x^2 + x - 2 = 0 \quad | \cdot 2$

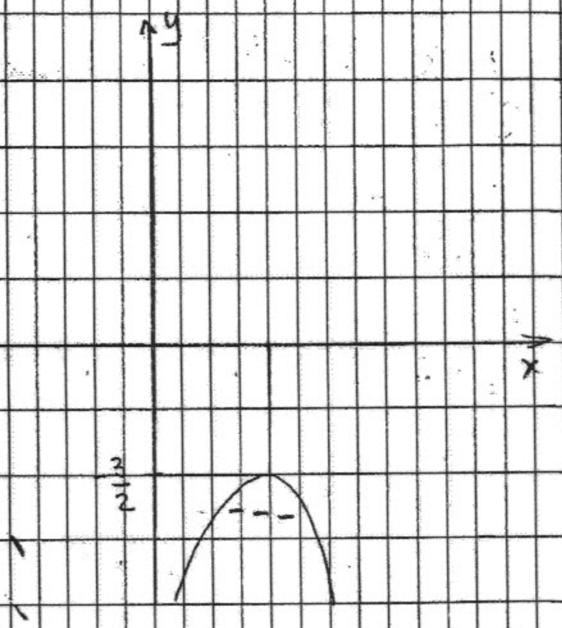
$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{-1}$

$D = -3$

$T\left(-\frac{1}{-1}, \frac{4-1}{-2}\right)$

$T\left(1, -\frac{3}{2}\right)$

$y < 0 \quad \forall x \in R$



3. $f(x) = x^2 - 4x + 3$

$g(x) = -x^2 + 6x - 8$

$h(x) = f(x) - g(x)$

$h(x) = 2x^2 - 10x + 11$

$x_{1,2} = \frac{10 \pm \overbrace{\sqrt{100-88}}^{\sqrt{12} = 2\sqrt{3}}}{4}$ $\nearrow x_1 = \frac{\cancel{2}(5+\sqrt{3})}{\cancel{4}_2} = \frac{5+\sqrt{3}}{2}$

$\searrow x_2 = \frac{\cancel{2}(5-\sqrt{3})}{\cancel{4}_2} = \frac{5-\sqrt{3}}{2}$

a) $f(x) > g(x)$

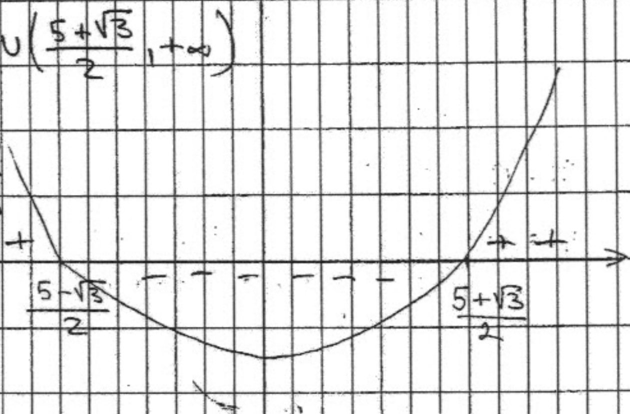
$h > 0 \Rightarrow f(x) > g(x)$ za $x \in \left(-\infty, \frac{5-\sqrt{3}}{2}\right) \cup \left(\frac{5+\sqrt{3}}{2}, +\infty\right)$

b) $f(x) = g(x)$

$h = 0 \Rightarrow f(x) = g(x)$ za $x \in \left\{\frac{5-\sqrt{3}}{2}, \frac{5+\sqrt{3}}{2}\right\}$

c) $f(x) < g(x)$

$h < 0 \Rightarrow f(x) < g(x)$ za $x \in \left(\frac{5-\sqrt{3}}{2}, \frac{5+\sqrt{3}}{2}\right)$



4. $y = x^2 + kx + 4$ - da dodiruje x-osu

$k = ?$

Uslov: $D = 0$

$k^2 - 16 = 0$

$k^2 = 16$

$k = \pm 4 \qquad k_1 = -4,\ k_2 = 4$

5. $mx^2 - 2mx + m - 3$ - da bude negativan $\forall x \in R$

I $m \neq 0$

$D = 4m^2 - 4m(m-3) = 4m^2 - 4m^2 + 12m = 12m$

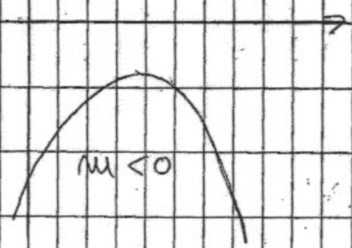
$D < 0 \ \forall m < 0 \quad a = m \quad m < 0 \Rightarrow a < 0$

$y < 0 \ \forall x \in R$

II $m = 0$

$mx^2 - 2mx + m - 3 = -3 < 0 \ \forall x \in R$

$\boxed{m \le 0}$ $y < 0 \ \forall x \in R$



6. $x^2 - (m-3)x - m + 6 > 0 \ \forall x \in R \quad m = ?$

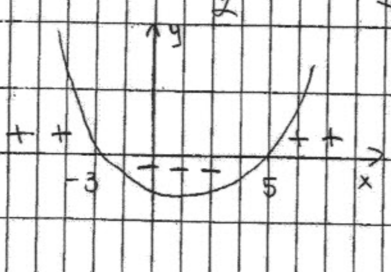
$a > 0 \Rightarrow y > 0 \ \forall x \in R$ ako je $D < 0$

$D = m^2 - 6m + 9 + 4(m-6) = m^2 - 6m + 9 + 4m - 24 = m^2 - 2m - 15$

$m^2 - 2m - 15 < 0$

$m_{1,2} = \frac{2 \pm \overbrace{\sqrt{4+60}}^{8}}{2} \to m_1 = 5, \quad \to m_2 = -3$

$\boxed{m \in (-3, 5)} \Rightarrow y > 0 \ \forall x \in R$



2.3. VIJETOVE FORMULE

$$x_1 + x_2 = -\frac{b}{a}, \quad x_1 x_2 = \frac{c}{a}$$

7. $mx^2 + 2(m-6)x + m - 3 = 0$

$D = 0$

$4(m-6)^2 - 4m \cdot (m-3) = 0$

$4(m^2 - 12m + 36) - 4m^2 + 12m = 0$

$\cancel{4m^2} - 48m + 144 - \cancel{4m^2} + 12m = 0$

$-36m = -144$

$m = 4$

$2x_1 = -\frac{b}{a}$

$2x_1 > 0$

$x_1 > 0$

8. $2x^2 - 3x + k - 1 = 0$

$D = 9 - 8(k-1) = 9 - 8k + 8 = 17 - 8k$

$D > 0 \Rightarrow k < \frac{17}{8}$

$x_1 x_2 = \frac{k-1}{2} \quad x_1 x_2 > 0$ za $k > 1$

$x_1 + x_2 = \frac{3}{2} \quad x_1 + x_2 > 0$ za $\forall k \in R$

Rešenja su pozitivna za $1 < k < \frac{17}{8}$

9. $x_1 = 2 \quad x_2 = 3$

$x_1 + x_2 = -\frac{b}{a}$

$x_1 x_2 = \frac{c}{a}$

$5 = -\frac{b}{a}$

$b = -5a$

$6 = \frac{c}{a}$

$c = 6a$

$ax^2 - 5ax + 6a = 0$

$a(x^2 - 5x + 6) = 0$

$\forall a \neq 0$

10. $x^2+(2+m-m^2)x-m^2=0$

$x_1+x_2=-\frac{b}{a}=\frac{m^2-m-2}{1}$ 3

$m_{1,2}=\frac{1\pm\sqrt{1+8}}{2} \to m_1=2$

$\to m_2=-1$

$m=-1 \quad x^2-2x-1=0 \Rightarrow D=4+4>0$

$m=2 \quad x^2-4x-4=0 \Rightarrow D=16+16>0$

$m\in\{-1,2\}$

11. $x^2-(2m-1)x+m^2+2=0$

$x_2=2x_1$

$x_1+x_2=3x_1$

$3x_1=-\frac{b}{a}=\frac{2m-1}{1}$

$x_1=\frac{2m-1}{3}$

$x_1x_2=2x_1^2$

$2x_1^2=\frac{c}{a}=\frac{m^2+2}{1}$

$x_1^2=\frac{m^2+2}{2}$

$\frac{(2m-1)^2}{9}=\frac{m^2+2}{2}$

$\frac{4m^2-4m+1}{9}=\frac{m^2+2}{2}$

$8m^2-8m+2=9m^2+18$

$-m^2-8m-16=0 \quad |\cdot(-1)$

$m^2+8m+16=0$

$m_{1,2}=\frac{-8\pm\sqrt{64-64}}{2}$ 0 $\boxed{m=-4}$

$x^2+9x+18=0$

$D=81-72=9>0$

12. $x^2-2x+m-3=0, \; m\in R \quad x_1^2+x_2^2=2$

$x_1+x_2=-\frac{b}{a}=2 \quad (x_1+x_2)^2=4 \quad x_1^2+2x_1x_2+x_2^2=4$

$x_1\cdot x_2=\frac{c}{a}=m-3$

$x_1^2+2(m-3)+x_2^2=4$

$2+2(m-3)=4$

$m-3=1$

$\boxed{m=4}$

13. $5x^2+mx-1=0$

$5x_1+2x_2=1 \Rightarrow x_2=\frac{1-5x_1}{2}$

$x_1+x_2=-\frac{b}{a}=-\frac{m}{5}$

$x_1x_2=\frac{c}{a}=-\frac{1}{5}$

$x_1\cdot\frac{1-5x_1}{2}=-\frac{1}{5}$

$\frac{x_1-5x_1^2}{2}=-\frac{1}{5}$

$5x_1-25x_1^2=-2$

$25x_1^2-5x_1-2=0$

$x_1=\frac{5\pm\sqrt{25+200}}{50}$ $\nearrow \frac{2}{5}$ $\searrow -\frac{1}{5}$ (15)

$x_2=\frac{1-5x_1}{2}=1$

$x_1+x_2=-\frac{m}{5}$

$m=-5(x_1+x_2)$

$m=-\cancel{5}\cdot\frac{4}{\cancel{5}}=-4$

$x_2=\frac{1-5x_1}{2}=-\frac{1}{2}$

$m=-5(x_1+x_2)=\frac{1}{2}$

$m\in\left\{-4,\frac{1}{2}\right\}$

14. $\frac{1}{x_1^3}+\frac{1}{x_2^3}=?$ $\quad 2x^2-3ax-2=0$

$\frac{1}{x_1^3}+\frac{1}{x_2^3}=\frac{x_2^3+x_1^3}{x_1^3\cdot x_2^3}=\frac{(x_1+x_2)(x_1^2-x_1x_2+x_2^2)}{(x_1x_2)^3}=$

$=\frac{(x_1-x_2)(x_1^2+2x_1x_2+x_2^2-3x_1x_2)}{(x_1x_2)^3}=\frac{(x_1+x_2)((x_1+x_2)^2-3x_1x_2)}{(x_1x_2)^3}=$

$=\frac{\frac{3a}{2}\left(\left(\frac{3a}{2}\right)^2-3(-1)\right)}{(-1)^3}=-\frac{3a}{2}\left(\frac{9a^2}{4}+3\right)=-\frac{3a}{2}\cdot\left(\frac{9a^2+12}{4}\right)=\frac{-27a^3-36a}{8}=$

$=\frac{-9a(3a^2+4)}{8}$

2.4 Nejednačine sa racionalnim funkcijama

15. a) $\dfrac{x^2-1}{x^2+x+1} < 1$

$$\frac{x^2-1}{x^2+x+1} - 1 < 0$$

$$\frac{\cancel{x^2}-1-\cancel{x^2}-x-1}{x^2+x+1} < 0$$

$$\frac{-x-2}{x^2+x+1} < 0$$

$$\left.\begin{matrix} D = 1-4 < 0 \\ a = 1 > 0 \end{matrix}\right\} \Rightarrow x^2+x+1 > 0,\ \forall x \in R$$

$$-x-2 < 0$$

$$-x < 2$$

$$\boxed{x > -2}$$

b) $\dfrac{1}{x^2-5x+6} \geq \dfrac{1}{2}$

$$\frac{1}{x^2-5x+6} - \frac{1}{2} \geq 0$$

$$\frac{1-\frac{x^2}{2}+\frac{5}{2}x-3}{x^2-5x+6} \geq 0 \quad \Big|\, 2$$

$$\frac{2-x^2+5x-6}{2(x^2-5x+6)} \geq 0$$

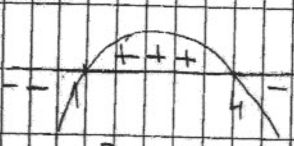
$$\frac{-x^2+5x-4}{2(x^2-5x+6)} \geq 0$$

$$\frac{(x-1)(x-4)}{2(x-3)(x-2)} \geq 0$$

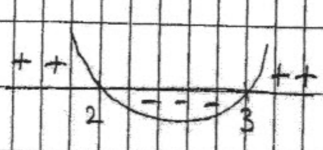
I $\quad x_{1,2} = \dfrac{-5 \pm \sqrt{25-16}^{\,3}}{-2}$ $\nearrow x_1 = 1$, $\searrow x_2 = 4$

II $\quad x_{1,2} = \dfrac{5 \pm \sqrt{25-24}^{\,1}}{2}$ $\nearrow x_1 = 3$, $\searrow x_2 = 2$

Rešenje: $x \in [1, 2) \cup (3, 4]$



$x \in [1, 4]$



$x \in (-\infty, 2) \cup (3, +\infty)$

c) $1 < \dfrac{3x^2-7x+8}{x^2+1} \leq 2$

$\dfrac{3x^2-7x+8}{x^2+1} - 1 > 0 \quad \wedge \quad \dfrac{3x^2-7x+8}{x^2+1} - 2 \leq 0$

$\dfrac{3x^2-7x+8-x^2-1}{x^2+1} > 0 \quad \wedge \quad \dfrac{3x^2-7x+8-2x^2-2}{x^2+1} \leq 0$

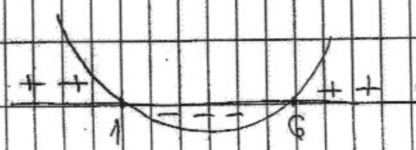
$\dfrac{2x^2-7x+7}{x^2+1} > 0 \quad \wedge \quad \dfrac{x^2-7x+6}{x^2+1} \leq 0$

$2x^2-7x+7>0 \qquad\qquad x^2-7x+6 \leq 0$

$x_{1,2} = \dfrac{7 \pm \sqrt{49-56}}{4} \notin R \qquad\qquad x_{1,2} = \dfrac{7 \pm \sqrt{\overset{5}{49-24}}}{2} \begin{array}{l} \nearrow x_1 = 6 \\ \searrow x_2 = 1 \end{array}$

$a = 2 > 0 \Rightarrow$

$\Rightarrow 2x^2-7x+7>0 \;\; \forall x \in R$



Rešenje: $x \in [1, 6]$

d) $\dfrac{x^2+4x-18}{5-x} \geq -2$

$\dfrac{x^2+4x-18}{5-x} + 2 \geq 0$

$\dfrac{x^2+4x-18+10-2x}{5-x} \geq 0$

$\dfrac{x^2+2x-8}{5-x} \geq 0$

$x_{1,2} = \dfrac{-2 \pm \sqrt{\overset{6}{4+32}}}{2} \begin{array}{l} \nearrow x_1 = 2 \\ \searrow x_2 = -4 \end{array}$

	$-\infty$		-4		2		5		$+\infty$
x^2+2x-8		+	0	−	0	+		+	
$5-x$		+		+		+	0	−	

Rešenje: $x \in (-\infty, -4] \cup [2, 5)$

e) $\dfrac{-x^2+2x-16}{x-6} \geq 3$

$\dfrac{-x^2+2x-16-3x+18}{x-6} \geq 0$

$\dfrac{-x^2-x+2}{x-6} \geq 0$

$x_{1,2} = \dfrac{1 \pm \sqrt{\overset{3}{1+8}}}{-2} \nearrow x_1 = -2, \searrow x_2 = 1$

	$-\infty$	-2		1		6	$+\infty$
$-x^2-x+2$	$-$	0	$+$	0	$-$		$-$
$x-6$	$-$		$-$		$-$	0	$+$
	$+$		$-$		$+$		$-$

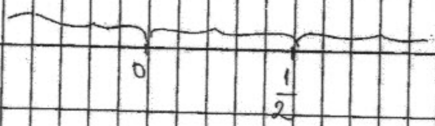
Rešenje: $x \in (-\infty, -2] \cup [1, 6)$

2.5. Jednačine i nejednačine sa apsolutnim vrednostima

16. $|2x-1| + |x| = 5$

I $2x-1 \geq 0$, $2x \geq 1$, $x \geq \frac{1}{2}$ $\qquad$ $2x-1 < 0$, $2x < 1$, $x < \frac{1}{2}$

II $x \geq 0$ $\qquad$ $x < 0$



$-2x+1-x=5$
$-3x=4$
$x=-\frac{4}{3}$ ✓

$-2x+1+x=5$
$-x=4$
$\cancel{x=-4}$

$2x-1+x=5$
$3x=6$
$x=2$ ✓

$x \in \left\{-\frac{4}{3}, 2\right\}$

17. $|x+2| < 2x-1$

I $x+2 \geq 0$, $x \geq -2$ $\qquad$ II $x+2 < 0$, $x < -2$

$x+2 < 2x-1$
$-x < -3$
$x > 3$ ✓

$-x-2 < 2x-1$
$-3x < 1$
$x > -\frac{1}{3}$

$x \in (3, +\infty)$

18. $|2x-5| \leq |x+4|$

I $2x-5 \geq 0$ $\quad 2x-5<0$ $\quad$ II $x+4 \geq 0$ $\quad x+4<0$

$2x \geq 5$ $\quad 2x<5$ $\quad x \geq -4$ $\quad x<-4$

$x \geq \frac{5}{2}$ $\quad x<\frac{5}{2}$

$-4 \qquad \frac{5}{2}$

$-2x+5 \leq -x-4$ $\qquad -2x+5 \leq x+4$ $\qquad 2x-5 \leq x+4$

$-x \leq -9$ $\qquad -3x \leq -1$ $\qquad x \leq 9$

$\cancel{x \geq 9}$ $\qquad x \geq \frac{1}{3}$ $\qquad x \in [\frac{5}{2}, 9]$

$x \in [\frac{1}{3}, \frac{5}{2})$

Rešenje: $\frac{1}{3} \leq x \leq 9$

19. $2x^2+|x-1|=2$

I $x-1 \geq 0$ $\quad$ II $x-1<0$

$x \geq 1$ $\quad x<1$

$2x^2+x-1=2$ $\qquad 2x^2-x+1=2$

$2x^2+x-3=0$ $\qquad 2x^2-x-1=0$

$x_{1,2}=\frac{-1 \pm \sqrt{1+24}}{4}$ $\nearrow \boxed{x_1=1}$ $\searrow x_2=-\frac{3}{2}$ $\qquad x_{1,2}=\frac{1 \pm \sqrt{1+8}}{4}$ $\nearrow x_1=1$ $\searrow \boxed{x_2=-\frac{1}{2}}$

Rešenje: $x \in \{-\frac{1}{2}, 1\}$

20. $|x^2-9|+x^2-4=5$

I $x^2-9 \geq 0$ $\qquad$ II $x^2-9<0$ $\qquad x^2-9+x^2-4=5$

$x^2 \geq 9$ $\qquad x^2<9$ $\qquad 2x^2-18=0$

$x \leq -3 \wedge x \geq 3$ $\qquad x>-3 \wedge x<3$ $\qquad x^2=9$

$\boxed{x=\pm 3}$

$-\cancel{x^2}+9+\cancel{x^2}-4=5$

$5=5$

$x\in(-3,3)$

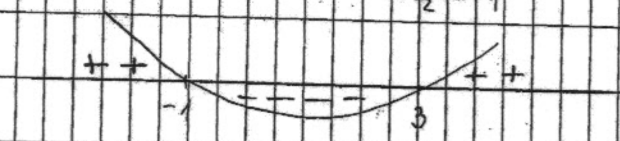
Rešenje: $x\in[-3,3]$

21. $|x^2-2x-3|<3x-3$

I $x^2-2x-3\geq 0$ $x\in(-\infty,-1]\cup[3,+\infty)$

II $x^2-2x-3<0$ $x\in(-1,3)$

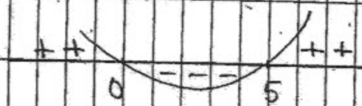
$x_{1,2}=\dfrac{2\pm\sqrt{\overset{4}{4+12}}}{2}$ $\nearrow x_1=3$, $\searrow x_2=-1$



$x^2-2x-3<3x-3$

$x^2-5x<0$

$x(x-5)<0$



$x\in(0,5)$

Uz uslov: $x\in[3,5)$

Rešenje: $x\in(2,5)$

$-x^2+2x+3<3x-3$

$-x^2-x+6<0$

$x_{1,2}=\dfrac{1\pm\sqrt{\overset{5}{1+24}}}{-2}$ $\nearrow x_1=-3$, $\searrow x_2=2$

$x\in(-\infty,-3)\cup(2,+\infty)$

Uz uslov: $x\in(2,3)$

2.6 Iracionalne jednacine

22. $(16-x^2)\sqrt{2x+6}=0$

Uslov: $2x+6\geq 0$
$2x\geq -6$
$x\geq -3$

$16-x^2=0$
$x^2=16$
$x=\pm 4$

$2x+6=0$
$2x=-6$
$x=-3$

Rešenje: $x\in\{-3,4\}$

23. $\sqrt{x-1}+\sqrt{2-x}=1$

I $x-1 \geq 0$ II $2-x \geq 0$

$x \geq 1$ $x \leq 2$

$\cancel{x}-1+2\sqrt{(x-1)(2-x)}+2-\cancel{x}=1$

+∞)

$2\sqrt{(x-1)(2-x)}=0$

$x-1=0$ $2-x=0$

$x=1$ $x=2$

Rešenje: $x \in \{1, 2\}$

24 $\sqrt{x-2}+|x-2|=4$

I $x-2 \geq 0$

$x \geq 2$

$\sqrt{x-2}+x-2=4$ Smena: $\sqrt{x-2}=t$

$t+t^2=4$

$t^2+t-4=0$

$t_{1,2}=\frac{-1 \pm \sqrt{1+16}}{2}$ $\nearrow t_1=\frac{-1+\sqrt{17}}{2}$ $\searrow t_2=\frac{-1-\sqrt{17}}{2}$

$\sqrt{x-2}=\frac{-1+\sqrt{17}}{2} \quad \uparrow^2$

$x-2=\frac{1-2\sqrt{17}+17}{4}$

$1-2\sqrt{17}+17=4x-8$

$4x=1-2\sqrt{17}+17+8$

$4x=26-2\sqrt{17}$

$x=\frac{26-2\sqrt{17}}{4}$

Rešenje: $x=\frac{13-\sqrt{17}}{2}$

2.7. Zadaci za vežbu

1. $x^2-(m+1)x-4=0$

Uslov: $D>0$

$D=b^2-4ac$

$D=(m+1)^2+16>0 \ \forall m\in R$

Rešenje: $m\in R$

2. $(2k-5)x^2-2(k-1)x+3=0$

Uslov: $D=0$

$D=b^2-4ac$

$D=4(k-1)^2-12(2k-5)=0$

$4k^2-8k+4-24k+60=0$

$4k^2-32k+64=0 \quad |:4$

$k^2-8k+16=0$

$k_{1,2}=\frac{8\pm\sqrt{\overset{0}{64-64}}}{2}$ Rešenje: $k=4$

3. $(k-2)x^2+2(k-2)x+2=0$

Uslov: $D<0$

$D=b^2-4ac$

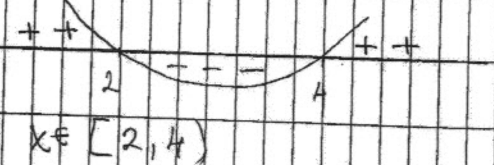
$4(k-2)^2-8(k-2)<0$

$4k^2-16k+16-8k+16<0$

$4k^2-24k+32<0 \quad |:4$

$k^2-6k+8<0$

$k_{1,2}=\frac{6\pm\sqrt{36-32}}{2}$ $\nearrow k_1=4$, $\searrow k_2=2$



$x\in(2,4)$

4. $x_1^3 + x_2^3 = ?$

$3x^2 - ax + 2a - 1 = 0$

$$x_1^3 + x_2^3 = (x_1 + x_2)(x_1^2 - x_1x_2 + x_2^2) = (x_1 + x_2)((x_1 + x_2)^2 - 3x_1x_2) =$$

$$= \frac{a}{3}\cdot\left(\left(\frac{a}{3}\right)^2 - 3\cdot\frac{2a-1}{3}\right) = \frac{a}{3}\left(\frac{a^2}{9} - \frac{6a-3}{3}\right) = \frac{a}{3}\left(\frac{a^2-18a+9}{9}\right) =$$

$$= \frac{a(a^2-18a+9)}{27}$$

5. $(5-x):(x+1) = (x-1):(x+a)$

$x_1^2 + x_2^2 = 26$

$(5-x)\cdot(x+a) = (x+1)\cdot(x-1)$

$5x + 5a - x^2 - xa = x^2 - 1$

$-2x^2 + 5x - xa + 5a - 1 = 0$

$-2x^2 + x(5-a) + 5a + 1 = 0$

$$x_{1,2} = \frac{-5+a \pm \sqrt{25-10a+a^2+40a+8}}{-4} \;\begin{cases} \nearrow \dfrac{a-5+\sqrt{a^2+30a+33}}{-4} \\ \searrow \dfrac{a-5-\sqrt{a^2+30a+33}}{-4} \end{cases}$$

$$\left(\frac{(a-5)+\sqrt{a^2+30a+33}}{-4}\right)^2 + \left(\frac{(a-5)-\sqrt{a^2+30a+33}}{-4}\right)^2 = 26$$

$$\frac{a^2-10a+25+\cancel{2(a-5)\sqrt{a^2+30a+33}}+a^2+30a+33}{16} +$$

$$+ \frac{a^2-10a+25-\cancel{2(a-5)\sqrt{a^2+30a+33}}+a^2+30a+33}{16} = 26$$

$$\frac{2a^2-20a+50+2a^2+60a+66}{16} = 26$$

$4a^2 + 40a + 116 = 416$

$4a^2 + 40a - 300 = 0$

$a^2 + 10a - 75 = 0$

$$a_{1,2} = \frac{-10 \pm \sqrt{100+300}^{\,20}}{2} \;\begin{cases} \nearrow a_1 = 5 \\ \searrow a_2 = -15 \end{cases}$$

6. $x^2 - \frac{15}{4}x + k = 0$

$x_1 = x_2^2$

$$x_{1,2} = \frac{\frac{15}{4} \pm \sqrt{\frac{225}{16} - 4k}}{2} \to x_1 = \frac{\frac{15}{4} + \sqrt{\frac{225}{16} - 4k}}{2}, \quad x_2 = \frac{\frac{15}{4} - \sqrt{\frac{225}{16} - 4k}}{2}$$

7.

$x^2 - mx - 4m = 0$ - realna i različita rešenja

$x_1^2 + x_2^2 < x_1 x_2 + 28$

$D > 0$

$D = b^2 - 4ac$

$D = m^2 + 4m$

$m^2 + 4m > 0$

$m(m+4) > 0$

$\boxed{m > 0} \vee m + 4 > 0$

$m > -4$

$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2$

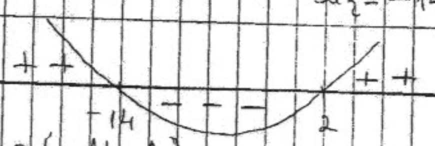
$\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} < \frac{c}{a} + 28$

$\left(\frac{m}{1}\right)^2 - 2 \cdot \frac{-4m}{1} < \frac{-4m}{1} + 28$

$m^2 + 8m < -4m + 28$

$m^2 + 12m - 28 < 0$

$$m_{1,2} = \frac{-12 \pm \sqrt{\overset{16}{144 + 112}}}{2} \to m_1 = 2, \quad m_2 = -14$$



$m \in (-14, 2)$

Rešenje: $m > 0 \cap m \in (-14, 2)$

$m \in (0, 2)$

8. $$\frac{7}{(x-2)(x-3)} + \frac{9}{x-3} < -1$$

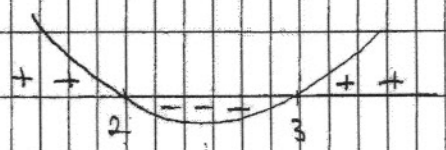
$$\frac{7 + 9x - 18}{(x-2)(x-3)} < -1$$

$$\frac{9x - 11 + (x-2)(x-3)}{(x-2)(x-3)} < 0$$

$$\frac{9x - 11 + x^2 - 3x - 2x + 6}{x^2 - 3x - 2x + 6} < 0$$

$$\frac{x^2 + 4x - 5}{x^2 - 5x + 6} < 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 + 20}}{2} \begin{cases} \nearrow x_1 = 1 \\ \searrow x_2 = -5 \end{cases} \qquad x_{1,2} = \frac{5 \pm \sqrt{25 - 24}}{2} \begin{cases} \nearrow x_1 = 3 \\ \searrow x_2 = 2 \end{cases}$$



Rešenje: $x \in (-5, 1) \cup (2, 3)$

9. $$\frac{x^2 - 5x + 6}{|x| + 7} < 0$$

$$x_{1,2} = \frac{5 \pm \sqrt{25 - 24}}{2} \begin{cases} \nearrow x_1 = 3 \\ \searrow x_2 = 2 \end{cases}$$

$x \in (2, 3)$

$|x| + 7 < 0$

$x > 0$: $x + 7 < 0$, $\cancel{x < -7}$

$x < 0$: $-x + 7 < 0$, $-x < -7$, $\cancel{x > 7}$

Rešenje: $x \in (2, 3)$

10. $(x^2-2x)(2x-2) \leq 9\dfrac{2x-2}{x^2-2x}$

$$(x^2-2x)(2x-2)-\frac{9(2x-2)}{x^2-2x} \leq 0$$

$$\frac{(x^2-2x)^2(2x-2)-9(2x-2)}{x^2-2x} \leq 0$$

$$\frac{((x^2-2x)^2-9)(2x-2)}{x^2-2x} \leq 0$$

$$\frac{(x^2-2x-3)(x^2-2x+3)(2x-2)}{x^2-2x} \leq 0$$

$x^2-2x-3=0$

$x_{1,2}=\dfrac{2\pm\sqrt{4+12}}{2}$ $\to x_1=3$, $\to x_2=-1$

~~$x^2-2x+3 \to$ uvek pozitivno~~

$2x-2=0$

$2x=2$

$x=1$

$x^2-2x=0$

$x(x-2)=0$

$x=0 \vee x=2$

	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,2)$	$(2,3)$	$(3,+\infty)$
x^2-2x-3	+	−	−	−	−	+
$2x-2$	−	−	−	+	+	+
x^2-2x	+	+	−	−	+	+
	−	+	−	+	−	+

$x \in (-\infty,-1] \cup (0,1] \cup (2,3]$

11. $|x| < |2+x|$

I $x \geq 0$, $x < 0$ II $2+x \geq 0$, $x \geq -2$; $2+x < 0$, $x < -2$

-2 0

$-x < -2-x$

$\cancel{0 < -2}$

$-x < 2+x$

$-2x < 2$

$x > -1$

$x \in (-1,0)$

$x < 2+x$

$0 < 2$

$x \geq 0$

Resenje: $x > -1$

12. $x^2 - 6mx + 9m^2 - 2m + 2 = 0$

$x_1 > 3 \quad \wedge \quad x_2 > 3$

$$x_{1,2} = \frac{6m \pm \sqrt{\overbrace{36m^2 - 4(9m^2 - 2m + 2)}^{-36m^2 + 8m - 8}}}{2} \nearrow \frac{6m + \sqrt{8m - 8}}{2} \searrow \frac{6m - \sqrt{8m - 8}}{2}$$

I

$$\frac{6m + \sqrt{8m-8}}{2} > 3$$

$6m + \sqrt{8m-8} > 6$

$\sqrt{8m-8} > 6 - 6m \quad /^2$

USLOV: $8m - 8 \geq 0$, $8m \geq 8$, $m \geq 1$

$8m - 8 > 36 - 72m + 36m^2$

$36m^2 - 80m + 44 < 0$

$$m_{1,2} = \frac{80 \pm \sqrt{6400 - 6336}}{72} \nearrow m_1 = \frac{88}{72} = 1\frac{16}{72} = 1\frac{2}{9} \searrow \boxed{m_2 = 1}$$

$m > 1$

II

$$\frac{6m - \sqrt{8m-8}}{2} > 3$$

$6m - \sqrt{8m-8} > 6$

$6m - 6 > \sqrt{8m-8} \quad /^2$

$36m^2 - 72m + 36 > 8m - 8$

$36m^2 - 80m + 44 > 0$

$m_{1,2} = \nearrow \boxed{m_1 = 1\frac{2}{9}} \searrow m_2 = 1$

$m > 1\frac{2}{9}$

Rešenje: $m > 1\frac{2}{9}$

13. $x^2+ax+b=0$

$x_1=a \quad x_2=b$

$x_{1,2}=\frac{-a\pm\sqrt{a^2-4b}}{2}$ $\nearrow x_1=\frac{-a+\sqrt{a^2-4b}}{2}$ $\searrow x_2=\frac{-a-\sqrt{a^2-4b}}{2}$

$\frac{-a+\sqrt{a^2-4b}}{2}=a$

$-a+\sqrt{a^2-4b}=2a$

$3a=\sqrt{a^2-4b}$

$\frac{-a-\sqrt{a^2-4b}}{2}=b$

$-a-\sqrt{a^2-4b}=2b$

$-a-3a=2b$

$-4a=2b$

$\boxed{b=-2a}$

$\frac{-a+\sqrt{a^2+8a}}{2}=a$

$-a+\sqrt{a^2+8a}=2a$

$\sqrt{a^2+8a}=3a \quad /^2$

$a^2+8a=9a^2$

$8a^2-8a=0$

$8a(a-1)=0$

$\boxed{\begin{matrix} a=0 & \vee & a=1 \\ b=0 & & b=-2 \end{matrix}}$

14. $x^2-8x+7=0$

$x_3=x_1+5 \quad x_4=x_2+5$ (6)

$x_{1,2}=\frac{8\pm\sqrt{64-28}}{2}$ $\nearrow x_1=7$ $\searrow x_2=1$

$x_3=12, \quad x_4=6$

$x_3+x_4=-\frac{b}{a} \qquad x_3x_4=\frac{c}{a}$

$18=-\frac{b}{a} \qquad 72=\frac{c}{a}$

$b=-18a \qquad c=72a$

$ax^2-18ax+72a=0$

$a(x^2-18x+72)=0$

$a\in R/\{0\}$

15. $mx^2-(m+2)x+m+2>0$

$D \leq 0$ za $m \geq 0$

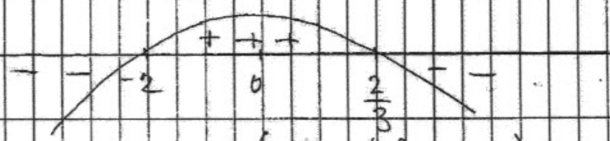
$D=b^2-4ac$

$(m+2)^2-4m(m+2) \leq 0$

$m^2+4m+4-4m^2-8m \leq 0$

$-3m^2-4m+4 \leq 0$

$$m_{1,2}=\frac{4\pm\overset{8}{\sqrt{16+48}}}{-6} \begin{array}{l} \nearrow m_1=-2 \\ \searrow m_2=\frac{2}{3} \end{array}$$



Rešenje: $m\in\{0\}\cup\left(\frac{2}{3},+\infty\right)$

16. $x^2+y^2=10$
$\underline{2x+y=5}$

$x^2+y^2=10$
$\underline{y=5-2x}$

$x^2+(5-2x)^2=10$
$\underline{y=5-2x}$

$x^2+25-20x+4x^2=10$
$\underline{y=5-2x}$

$5x^2-20x+15=0$
$\underline{y=5-2x}$

$$x_{1,2}=\frac{20\pm\overset{10}{\sqrt{400-300}}}{10} \begin{array}{l} \nearrow x_1=3 \\ \searrow x_2=1 \end{array}$$

$x_1=3 \quad x_2=1$

$y_1=-1 \quad y_2=3$

$(x,y)\in\{(1,3),(3,-1)\}$

17. $\sqrt{\frac{x-1}{2x+1}} > 1$

Uslov: $\frac{x-1}{2x+1} \geq 0$

$x-1=0$
$x=1$

$2x+1=0$
$2x=-1$
$x=-\frac{1}{2}$

	$-\infty$ … $-\frac{1}{2}$	$-\frac{1}{2}$ … 1	1 … $+\infty$
$x-1$	$-$	$-$	$+$
$2x+1$	$-$	$+$	$+$
	$+$	$-$	$+$

$x \in \left(-\infty, -\frac{1}{2}\right) \cup \left[1, +\infty\right)$

$\frac{x-1}{2x+1} > 1$

$\frac{x-1-2x-1}{2x+1} > 0$

$\frac{-x-2}{2x+1} > 0$

$-x-2=0$
$-x=2$
$x=-2$

	$-\infty$ … -2	-2 … $-\frac{1}{2}$	$-\frac{1}{2}$ … $+\infty$
$-x-2$	$+$	$-$	$-$
$2x+1$	$-$	$-$	$+$
	$-$	$+$	$-$

R: $x \in \left(-2, -\frac{1}{2}\right)$

18. $-5x^2+bx+c=0$

$x_1 x_2 = \frac{c}{a} = 12$

$b, c \in R$

$T(x, y)$

$x = -\frac{b}{2a} = 4$

$x_{1,2} = ?$

$\frac{c}{a} = 12$

$c = 12a = -60$

$-\frac{b}{2a} = 4$

$b = -8a = 40$

$x_{1,2} = \frac{-40 \pm \overset{20}{\sqrt{1600-1200}}}{-10}$

$x_1 = 2$

$x_2 = 6$

19. $kx^2-2k^2x+k^3+k^2-3k-4<0 \quad \forall x\in R$

$D<0 \quad \boxed{k<0}$

$D=b^2-4ac$

$4k^4-4k(k^3+k^2-3k-4)<0$

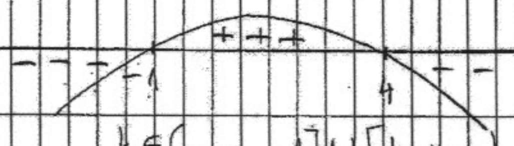
$\cancel{4k^4}-\cancel{4k^4}-4k^3+12k^2+16k<0$

$-4k^3+12k^2+16k<0$

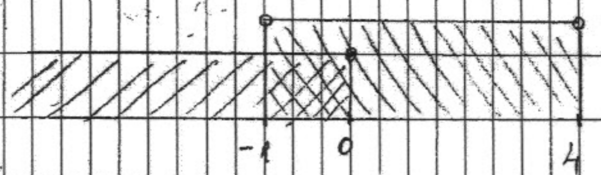
$k(-4k^2+12k+16)<0$

$k\leq 0 \Rightarrow -4k^2+12k+16>0$

$k_{1,2}=\dfrac{-12\pm\sqrt{144+256}}{-8} \to k_1=4, \quad k_2=-1$



$k\in(-\infty,-1]\cup[4,+\infty)$



Rešenje $k\in(-1,0]$

28. $\dfrac{2x^2-2x-1}{2x+1}\leq 4$

$\dfrac{2x^2-2x-1-8x-4}{2x+1}\leq 0$

$\dfrac{2x^2-10x-5}{2x+1}\leq 0$

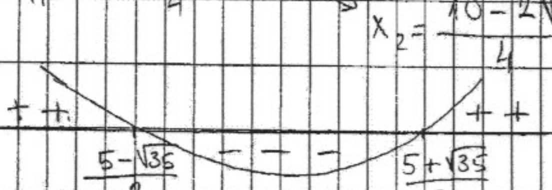
$2x^2-10x-5\leq 0$

$x_{1,2}=\dfrac{10\pm\sqrt{100+40}}{4} \to x_1=\dfrac{10+2\sqrt{35}}{4}=\dfrac{5+\sqrt{35}}{2}, \quad x_2=\dfrac{10-2\sqrt{35}}{4}=\dfrac{5-\sqrt{35}}{2}$

$2x+1<0$

$2x<-1$

$x<-\dfrac{1}{2}$



Rešenje: $x\in\left(-\infty,-\frac{1}{2}\right)\cup\left[\frac{5-\sqrt{35}}{2},\frac{5+\sqrt{35}}{2}\right]$

29. $a \in R \quad b \in R$

$ax^2 - x + b = 0$

$x = x_0$

$2x_0 = x_0^2$

$D = 0$

$1 - 4ab = 0$

$4ab = 1$

$\boxed{ab = \frac{1}{4}}$

$-\frac{-1}{a} = \frac{b}{a}$

$\frac{1}{a} = \frac{b}{a}$

$a = ab$

$a(b-1) = 0$

$a_2 = 0 \quad b - 1 = 0$

$b_1 = 1 \quad a_1 = \frac{1}{4}$

3. Eksponencijalni izrazi i logaritmi

3.1. Eksponencijalne jednačine i nejednačine

↙ eksponencijalna jednačina

$a^x = b \quad a, b \in R \quad a, b > 0 \quad a \neq 1$

$y = a^x$, $a > 0$ i $a \neq 1 \Rightarrow$ eksponencijalna funkcija

$a > 1$ – funkcija je monotono rastuća, pa je

$a^{f(x)} \geq a^{g(x)} \iff f(x) \geq g(x)$

$0 < a < 1 \Rightarrow$ funkcija je monotono opadajuća, pa je

$a^{f(x)} \geq a^{g(x)} \iff f(x) \leq g(x)$

* Za eksponencijalnu funkciju važi:

$a^x > 0 \ \forall x \in R \quad a^x = a^y \iff x = y$

Specijalno je: $a^x = 1 \iff x = 0$