1 Operacije sa algebarskim izrazima

Oznake za skupove brojeva:

N – prirodni brojevi $N = \{1, 2, 3, \dots\}$

Z – celi brojevi $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$

Q – racionalni brojevi (brojevi koji se mogu predstaviti u obliku razlomka

$Q = \left\{ \frac{p}{q} \mid p \in Z \land q \in N \right\}$

I – iracionalni brojevi (brojevi koji se ne mogu predstaviti u obliku razlomka

R – realni brojevi (svi racionalni i iracionalni brojevi) $R = Q \cup I$

C – kompleksni brojevi (brojevi oblika $a + bi$, gde je $a, b \in R$ i $i^2 = -1$)

1.1. Stepenovanje i korenovanje

- Stepen čiji je izložilac ceo broj:

$a^1 = a$, $a^{n+1} = a \cdot a^n$ za $n \in N$, $a \in R$

$a^0 = 1$, $a \neq 0$, $a^{-n} = \frac{1}{a^n}$, $a \neq 0$, $n \in N$

* n-ti koren broja (stepen čiji je izložilac racionalan broj):

- za $a \geq 0$, $a \in R$, $n \in N$, n-ti realni koren broja a je nenegativno rešenje jednačine $x^n = a$ u skupu R i označavamo ga sa $\sqrt[n]{a}$ ili $a^{\frac{1}{n}}$.
- za $a < 0$, $a \in R$ i n je neparan broj, n-ti realni koren broja a je rešenje jednačine $x^n = a$ u skupu R i označavamo ga sa $\sqrt[n]{a}$ ili $a^{\frac{1}{n}}$.

$(\sqrt[n]{a})^n = a$ $\forall a$ za koje je $\sqrt[n]{a}$ definisan.

$$\sqrt{a^2} = |a| = \begin{cases} a & \text{za } a \geq 0 \\ -a & \text{za } a < 0 \end{cases}$$

- Pravila za računanje sa stepenima čiji je izložilac racionalan broj:

$(-a)^n = (-1)^n \cdot a^n$

$$(-a)^n = \begin{cases} a^n, & n = 2k \\ -a^n, & n = 2k+1 \end{cases} \quad \text{za } a \in R,\ k \in N$$

$a^n \cdot a^k = a^{n+k}$

$a^n : a^k = \frac{a^n}{a^k} = a^{n-k}$

$(a^n)^k = a^{n \cdot k}$ $\qquad$ $\left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$(a \cdot b)^n = a^n \cdot b^n$ $\qquad$ $\sqrt[n]{a^k} = a^{\frac{k}{n}}$

$(a+b)^n \neq a^n + b^n$ i $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

1.

a) $\left(\frac{x^5 \cdot x^4}{x^6}\right)^2 = \left(\frac{x^9}{x^6}\right)^2 = (x^3)^2 = x^6$

$x \neq 0$

b) $(4x^{-2})^3 \cdot 2x^{12} : (16x^{-3})^2 = 4^3 \cdot x^{-6} \cdot 2x^{12} : (4^2 x^{-3})^2 =$

$x \neq 0$

$= 2^6 \cdot x^{-6} \cdot 2x^{12} : (2^4 \cdot x^{-3})^2 =$

$= 2^7 \cdot x^6 : (2^8 \cdot x^{-6}) =$

$= 2^{-1} \cdot x^{12} = \frac{x^{12}}{2}$

c) $\left(-\frac{3}{4}\right)^{-2} \cdot \left(\frac{2}{3}\right)^{-4} = \left(-\frac{3}{2^2}\right)^{-2} \cdot \left(\frac{3}{2}\right)^{-4} = \left(-\frac{2^2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^4 =$

$= \frac{2^4}{3^2} \cdot \frac{2^4}{3^4} = \frac{2^8}{3^6}$

d) $(x^{-3})^2 \cdot (x^{-5})^{-1} = x^{-6} \cdot x^5 = x^{-1} = \frac{1}{x}$

e) $(p^2x^{-3})^{-2} \cdot (p^{-1}\cdot x^2)^{-3} = p^{-4}x^6 \cdot p^3x^{-6} = p^{-1}x^0 = \frac{1}{p}$

f) $(a^{-3}b^{-1})^{-1} : (a^{-2}b^3)^{-2} = (a^3b^1) : (a^4b^{-6}) = a^{-1}b^7 = \frac{b^7}{a}$

g) $(x-y)^{-2} \cdot (x-y)^0 : (x-y) = (x-y)^{-2} : (x-y)^1 = (x-y)^{-3} = \frac{1}{(x-y)^3}$

h) $\frac{2}{3^{-2}} \cdot \frac{2^{-2}}{3} : \left(-\frac{2}{3}\right)^{-2} = \frac{2^{-1}}{3^{-1}} : \left(-\frac{3}{2}\right)^2 = \frac{3}{2} \cdot \frac{2^2}{3^2} = \frac{2}{3}$

2.

a) $\sqrt[5]{x^2\sqrt{x}} = \left(x^2 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{5}} = \left(x^{\frac{5}{2}}\right)^{\frac{1}{5}} = x^{\frac{1}{2}} = \sqrt{x}$

b) $\sqrt[3]{x^6y} : \sqrt[4]{x^3y^{12}} = (x^6y)^{\frac{1}{3}} : (x^3y^{12})^{\frac{1}{4}} = \left(x^2y^{\frac{1}{3}}\right) : \left(x^{\frac{3}{4}}y^3\right) = x^{\frac{5}{4}}y^{-\frac{8}{3}} =$
$= x^{\frac{15}{12}}y^{-\frac{32}{12}} = \sqrt[12]{\frac{x^{15}}{y^{32}}}$

c) $a^{\frac{3}{4}} \cdot a^{\frac{5}{6}} : a^{\frac{1}{8}} = a^{\left(\frac{3}{4}+\frac{5}{6}-\frac{1}{8}\right)} = a^{\left(\frac{18}{24}+\frac{20}{24}-\frac{3}{24}\right)} = a^{\frac{35}{24}} = \sqrt[24]{a^{35}}$

d) $\sqrt[6]{x^5} \cdot \sqrt[8]{x} : \sqrt[3]{x^{10}} = x^{\frac{5}{6}} \cdot x^{\frac{1}{8}} : x^{\frac{10}{3}} = x^{\frac{23}{24}} : x^{\frac{80}{24}} = x^{-\frac{57}{24}} = \frac{1}{\sqrt[8]{x^{19}}}$

e) $\sqrt{\frac{x+y}{x-y}} \cdot \sqrt[3]{\left(\frac{x+y}{x-y}\right)^2} = \left(\frac{x+y}{x-y}\right)^{\frac{1}{2}} \cdot \left(\frac{x+y}{x-y}\right)^{\frac{2}{3}} = \left(\frac{x+y}{x-y}\right)^{\frac{7}{6}} = \sqrt[6]{\left(\frac{x+y}{x-y}\right)^7}$

f) $\left(a^{\frac{3}{4}} - a^{\frac{5}{6}}\right) : \sqrt{a} = \left(a^{\frac{3}{4}} - a^{\frac{5}{6}}\right) \cdot a^{-\frac{1}{2}} = a^{\frac{1}{4}} - a^{\frac{2}{6}} = \sqrt{a} - \sqrt[3]{a}$

g) $\sqrt[8]{a^5b^7} \cdot \sqrt[4]{a \cdot b^3} : \sqrt{a^3b^4} = (a^5b^7)^{\frac{1}{8}} \cdot (a \cdot b^3)^{\frac{1}{4}} : (a^3b^4)^{\frac{1}{2}} =$
$= \left(a^{\frac{5}{8}}b^{\frac{7}{8}} \cdot a^{\frac{1}{4}}b^{\frac{3}{4}}\right) : \left(a^{\frac{3}{2}}b^2\right) = \left(a^{\frac{7}{8}}b^{\frac{13}{8}}\right) : \left(a^{\frac{12}{8}}b^{\frac{16}{8}}\right) = a^{-\frac{5}{8}}b^{-\frac{3}{8}} =$
$= \frac{1}{\sqrt[8]{a^5b^3}}$

1.2 Algebarski izrazi

$ax + ay = a(x+y)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a-b)(a+b)$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

$a - b = (\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2})$

3.

a) $\dfrac{a^2-9}{2a^2+12a+18} = \dfrac{(a-3)(a+3)}{2(a^2+6a+9)} = \dfrac{(a-3)\cancel{(a+3)}}{2(a+3)^{\cancel{2}}} = \dfrac{(a-3)}{2(a+3)}$

b) $\dfrac{6x^2-12x+6}{x^3-1} = \dfrac{6(x^2-2x+1)}{(x-1)(x^2+x+1)} = \dfrac{6(x-1)^{\cancel{2}}}{\cancel{(x-1)}(x^2+x+1)} = \dfrac{6(x-1)}{x^2+x+1}$

c) $\dfrac{a^2+b^2+2ab-c^2}{a^2+c^2+2ac-b^2} = \dfrac{(a+b)^2-c^2}{(a+c)^2-b^2} = \dfrac{\cancel{(a+b+c)}(a+b-c)}{\cancel{(a+c+b)}(a+c-b)} = \dfrac{a+b-c}{a-b+c}$

d) $\dfrac{2x^2-2x}{1-x^2} = \dfrac{2x(x-1)}{(1-x)(1+x)} = \dfrac{2x\cancel{(x-1)}}{-\cancel{(x-1)}(x+1)} = \dfrac{-2x}{x+1}$

e) $\dfrac{3a^2b-a^3}{9ab^2-6a^2b+a^3} = \dfrac{a^{\cancel{2}}(3b-a)}{\cancel{a}(9b^2-6ab+a^2)} = \dfrac{a\cancel{(3b-a)}}{(3b-a)^{\cancel{2}}} = \dfrac{a}{3b-a}$

f) $\frac{3x^2+4xy}{9x^2y-16y^3}=\frac{x(3x+4y)}{y(9x^2-16y^2)}=\frac{x\cancel{(3x+4y)}}{y(3x-4y)\cancel{(3x+4y)}}=\frac{x}{y(3x-4y)}$

g) $\frac{x^2-4x+4}{3x^2-12}=\frac{(x-2)^2}{3(x^2-4)}=\frac{(x-2)^{\cancel{2}}}{3\cancel{(x-2)}(x+2)}=\frac{x-2}{3(x+2)}$

h) $\frac{(x+y)^2-z^2}{x^2-(y+z)^2}=\frac{\cancel{(x+y+z)}(x+y-z)}{(x-y-z)\cancel{(x+y+z)}}=\frac{x+y-z}{x-y-z}$

i) $\frac{a^3+1}{a^3+3a^2+3a+1}=\frac{\cancel{(a+1)}(a^2-a+1)}{(a+1)^{\cancel{3}\,2}}=\frac{a^2-a+1}{(a+1)^2}$

4.

a) $\left(a+\frac{ab}{a-b}\right)\left(\frac{ab}{a+b}-a\right):\frac{a^2+b^2}{a^2-b^2}=$

$=\left(\frac{a^2-\cancel{ab}+\cancel{ab}}{a-b}\right)\left(\frac{\cancel{ab}-a^2-\cancel{ab}}{a+b}\right)\cdot\frac{(a-b)(a+b)}{a^2+b^2}=$

$\frac{-a^4}{\cancel{(a-b)(a+b)}}\cdot\frac{\cancel{(a-b)(a+b)}}{a^2+b^2}=-\frac{a^4}{a^2+b^2}$

b) $\frac{a^2-b^2}{a-b}-\frac{a^3-b^3}{a^2-b^2}=\frac{(a-b)(a+b)}{a-b}-\frac{(a-b)(a^2+ab+b^2)}{(a-b)(a+b)}=$

$\frac{(a-b)(a+b)^2-(a-b)(a^2+ab+b^2)}{(a-b)(a+b)}=\frac{\cancel{(a-b)}(\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}-ab-\cancel{b^2})}{\cancel{(a-b)}(a+b)}=$

$=\frac{ab}{a+b}$

c) $\left(\frac{1}{a+b}-\frac{a}{(a+b)^2}\right):\left(\frac{1}{a+b}-\frac{a}{a^2-b^2}\right)=$

$$= \frac{\cancel{a}+\cancel{b}-\cancel{a}}{(a+b)^{\cancel{2}}} \cdot \frac{(a-b)\cancel{(a+b)}}{\cancel{a}-\cancel{b}-\cancel{a}} = -\frac{a-b}{a+b} = \frac{b-a}{a+b}$$

d) $$\frac{a^2-ab+b^2}{a^2-b^2} \cdot \left(\frac{a-b}{a+b} - \frac{a^3-b^3}{a^3+b^3}\right) =$$

$$= \frac{a^2-ab+b^2}{(a-b)(a+b)} \cdot \left(\frac{a-b}{a+b} - \frac{(a-b)(a^2+ab+b^2)}{(a+b)(a^2-ab+b^2)}\right)$$

$$= \frac{a^2-ab+b^2}{(a-b)(a+b)} \cdot \frac{(a-b)(a^2-ab+b^2)-(a-b)(a^2+ab+b^2)}{(a+b)(a^2-ab+b^2)} =$$

$$= \frac{\cancel{a^2-ab+b^2}}{\cancel{(a-b)}(a+b)} \cdot \frac{\cancel{(a-b)}(\cancel{a^2}-ab+\cancel{b^2}-\cancel{a^2}-ab-\cancel{b^2})}{(a+b)\cancel{(a^2-ab+b^2)}} = \frac{-2ab}{(a+b)^2}$$

e) $$\left(\frac{4(a+b)^2}{ab} - 16\right) \cdot \frac{(a+b)^2-ab}{ab} : \frac{a^3-b^3}{ab} =$$

$$= \frac{4a^2+8ab+4b^2-16ab}{ab} \cdot \frac{a^2+2ab+b^2-ab}{ab} \cdot \frac{ab}{(a-b)(a^2+ab+b^2)} =$$

$$= \frac{(4a^2-8ab+4b^2)\cancel{(a^2+ab+b^2)}}{(ab)^{\cancel{2}}} \cdot \frac{\cancel{ab}}{(a-b)\cancel{(a^2+ab+b^2)}} =$$

$$= \frac{4(a-b)^{\cancel{2}}}{ab} \cdot \frac{1}{\cancel{(a-b)}} = \frac{4(a-b)}{ab}$$

f) $$\frac{1}{\frac{a}{a-2b} - \frac{2b}{a+2b}} \cdot \frac{a^2+4b^2}{a^2-4b^2} = \frac{1}{\frac{a(a+2b)-2b(a-2b)}{(a-2b)(a+2b)}} \cdot \frac{a^2+4b^2}{(a-2b)(a+2b)} =$$

$$= \frac{\cancel{(a-2b)(a+2b)}}{\cancel{a^2}+\cancel{2ab}-\cancel{2ab}+\cancel{4b^2}} \cdot \frac{\cancel{a^2+4b^2}}{\cancel{(a-2b)(a+2b)}} = 1$$

g) $\dfrac{36a-108}{10a^3-270} : \left(\dfrac{a+3}{a-3} + \dfrac{a-3}{a+3} - 2\right) =$

$= \dfrac{36(a-3)}{10(a^3-27)} : \left(\dfrac{(a+3)^2+(a-3)^2}{(a-3)(a+3)} - 2\right) =$

$= \dfrac{36(a-3)}{10(a-3)(a^2+3a+9)} : \left(\dfrac{\cancel{a^2}+\cancel{6a}+9+\cancel{a^2}-\cancel{6a}+9-\cancel{2a^2}+18}{(a-3)(a+3)}\right) =$

$= \dfrac{\cancel{36}\cancel{(a-3)}}{10\cancel{(a-3)}(a^2+3a+9)} \cdot \dfrac{(a-3)(a+3)}{\cancel{36}} = \dfrac{(a-3)(a+3)}{10(a^2+3a+9)}$

h) $\left(\sqrt{8-2\sqrt7} - \sqrt{8+2\sqrt7}\right)^2 = 8 - \cancel{2\sqrt7} - 2\sqrt{8-2\sqrt7}\sqrt{8+2\sqrt7} + 8 + \cancel{2\sqrt7} =$

$= 16 - 2\sqrt{64-28} = 16 - 2\sqrt{36} = 16 - 12 = 4$

i) $\left(a + \sqrt{a^2-b^2} - \dfrac{b^2-a}{a-\sqrt{a^2-b^2}}\right) : \sqrt{a^4-a^2b^2} =$

$= \dfrac{a^2-(a^2-b^2)-(b^2-a)}{a-\sqrt{a^2-b^2}} \cdot \dfrac{1}{a\sqrt{a^2-b^2}}$

$\dfrac{\cancel{a^2}-\cancel{a^2}+\cancel{b^2}-\cancel{b^2}+\cancel{a}}{a-\sqrt{a^2-b^2}} \cdot \dfrac{1}{\cancel{a}\sqrt{a^2-b^2}} = \dfrac{1}{a\sqrt{a^2-b^2}-(a^2-b^2)} = \dfrac{1}{a\sqrt{a^2-b^2}-a^2+b^2}$

4. $\left(\dfrac{a^{\frac32}+b^{\frac32}}{a^{\frac12}+b^{\frac12}} - \sqrt{ab}\right) : \left(\dfrac{\sqrt a+\sqrt b}{a-b}\right)^{-2} =$

$= \left(\dfrac{(\sqrt a)^3+(\sqrt b)^3}{\sqrt a+\sqrt b} - \sqrt{ab}\right) : \left(\dfrac{(\sqrt a-\sqrt b)(\sqrt a+\sqrt b)}{\sqrt a+\sqrt b}\right)^2 =$

$= \left(\dfrac{\cancel{(\sqrt a+\sqrt b)}(a-\sqrt{ab}+b)}{\cancel{\sqrt a+\sqrt b}} - \sqrt{ab}\right) \cdot \dfrac{\cancel{(\sqrt a+\sqrt b)^2}}{(\sqrt a-\sqrt b)^2\cancel{(\sqrt a+\sqrt b)^2}} =$

$= \dfrac{a-\sqrt{ab}+b-\sqrt{ab}}{a-2\sqrt{ab}+b} = \dfrac{a-2\sqrt{ab}+b}{a-2\sqrt{ab}+b} = 1$

5.

$$\frac{\sqrt{a^2-2ab+b^2}}{\sqrt{a^2+2ab+b^2}}+\frac{2a}{a+b} \qquad \text{a) } 0<b<a \qquad \text{b) } 0<a<b$$

a) $$\frac{\sqrt{(a-b)^2}}{\sqrt{(a+b)^2}}+\frac{2a}{a+b}=\frac{a-b}{a+b}+\frac{2a}{a+b}=\frac{a-b+2a}{a+b}=\frac{3a-b}{a+b}$$

b) $$\frac{\sqrt{(a-b)^2}}{\sqrt{(a+b)^2}}+\frac{2a}{a+b}=\frac{|a-b|}{a+b}+\frac{2a}{a+b}=\frac{b-a+2a}{a+b}=\frac{b+a}{a+b}=1$$

6. $m>n>0$

$$\left(\frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}-\frac{m-\sqrt{m^2-n^2}}{m+\sqrt{m^2-n^2}}\right)\cdot\frac{n^2}{4m\sqrt{m^2-n^2}}=$$

$$=\frac{\left(m+\sqrt{m^2-n^2}\right)^2-\left(m-\sqrt{m^2-n^2}\right)^2}{\left(m-\sqrt{m^2-n^2}\right)\left(m+\sqrt{m^2-n^2}\right)}\cdot\frac{n^2}{4m\sqrt{m^2-n^2}}=$$

$$=\frac{\cancel{m^2}+2m\sqrt{m^2-n^2}+\cancel{m^2}-\cancel{n^2}-\cancel{m^2}+2m\sqrt{m^2-n^2}-\cancel{m^2}+\cancel{n^2}}{\cancel{m^2}-\cancel{m^2}+n^2}\cdot\frac{n^2}{4m\sqrt{m^2-n^2}}$$

$$=\frac{4m\sqrt{m^2-n^2}}{n^2}\cdot\frac{n^2}{4m\sqrt{m^2-n^2}}=1$$

7. $a>0,\ b>0$

$$a\left(\frac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1}+b\left(\frac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}=a\,\frac{2b\sqrt{a}}{\sqrt{a}+\sqrt{b}}+b\,\frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}=$$

$$=\frac{2ba\sqrt{a}+2ab\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{2ab\cancel{(\sqrt{a}+\sqrt{b})}}{\cancel{\sqrt{a}+\sqrt{b}}}=2ab$$

8. $x \neq \pm y$

$$\frac{x^3+y^3}{x+y} : (x^2-y^2) + \frac{2y}{x+y} - \frac{xy}{x^2-y^2} =$$

$$= \frac{\cancel{(x+y)}(x^2-xy+y^2)}{x+y} \cdot \frac{1}{(x-y)\cancel{(x+y)}} + \frac{2y}{x+y} - \frac{xy}{(x+y)(x-y)} =$$

$$= \frac{x^2-xy+y^2+2y(x-y)-xy}{(x+y)(x-y)} = \frac{x^2-\cancel{2xy}+\cancel{2xy}-2y^2+y^2}{x^2-y^2} = \frac{x^2-y^2}{x^2-y^2} = 1$$

9. $$3\left(\frac{2}{5+\sqrt{10}} - \frac{7}{\sqrt{10}} + \frac{5}{\sqrt{10}-2}\right) =$$

$$= 3\left(\frac{2\sqrt{10}(\sqrt{10}-2)-7(5+\sqrt{10})(\sqrt{10}-2)+5\sqrt{10}(5+\sqrt{10})}{\sqrt{10}(5+\sqrt{10})(\sqrt{10}-2)}\right) =$$

$$= 3 \cdot \frac{20-4\sqrt{10}-7(5\sqrt{10}-\cancel{10}+\cancel{10}-2\sqrt{10})+25\sqrt{10}+50}{\sqrt{10}(5\sqrt{10}-\cancel{10}+\cancel{10}-2\sqrt{10})} =$$

$$= \cancel{3} \cdot \frac{70+\cancel{21\sqrt{10}}-\cancel{21\sqrt{10}}}{\cancel{30}_{10}} = \frac{70}{10} = 7$$

10. $$\sqrt[4]{2} + \frac{1}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}} - \sqrt[3]{3} = \sqrt[4]{2} + \frac{1}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}} \cdot \frac{\sqrt[3]{3}-\sqrt[3]{2}}{\sqrt[3]{3}-\sqrt[3]{2}} - \sqrt[3]{3} =$$

$$= \sqrt[4]{2} + \frac{\sqrt[3]{3}-\sqrt[3]{2}}{\sqrt[3]{27}-\cancel{\sqrt[3]{18}}+\cancel{\sqrt[3]{18}}-\cancel{\sqrt[3]{12}}+\cancel{\sqrt[3]{12}}-\sqrt[3]{8}} - \sqrt[3]{3} =$$

$$= \sqrt[4]{2} + \frac{\sqrt[3]{3}-\sqrt[3]{2}}{3-2} - \sqrt[3]{3} = \sqrt[4]{2} + \sqrt[3]{3} - \sqrt[3]{2} - \sqrt[3]{3} = 0$$

11. $$\sqrt[3]{20-14\sqrt{2}} + \sqrt[3]{20+14\sqrt{2}} = \sqrt{8-12\sqrt{2}+12-2\sqrt{2}} + \sqrt[3]{8+12\sqrt{2}+12+2\sqrt{2}} =$$

$$= \sqrt[3]{(2-\sqrt{2})^3} + \sqrt[3]{(2+\sqrt{2})^3} = 2-\cancel{\sqrt{2}}+2+\cancel{\sqrt{2}} = 4$$

1.3. Zadaci za vežbu

1. $$\frac{2^{-3}}{3^{2}}\cdot\left(\frac{2^{-2}}{3}\right)^{-2}:2=\frac{2^{-3}\cdot 2^{4}\cdot 2^{-1}}{3^{2}\cdot 3^{-2}}=\frac{2^{0}}{3^{0}}=1$$

2. $$\left(a-\frac{27}{a^{2}}\right):\frac{a^{2}+3a+9}{a^{2}}=\frac{a^{3}-27}{\cancel{a^{2}}}\cdot\frac{\cancel{a^{2}}}{a^{2}+3a+9}=$$

$$=\frac{(a-3)\cancel{(a^{2}+3a+9)}}{\cancel{a^{2}+3a+9}}=a-3$$

3. $$\frac{x}{y}\left(1-\frac{x}{x+y}\right)+\left(\frac{x}{y}\right)^{-1}\left(1-\frac{y}{x+y}\right)=$$

$$=\frac{x}{y}\left(\frac{\cancel{x}+y-\cancel{x}}{x+y}\right)+\frac{y}{x}\left(\frac{x+\cancel{y}-\cancel{y}}{x+y}\right)=$$

$$=\frac{x}{\cancel{y}}\cdot\frac{\cancel{y}}{x+y}+\frac{y}{\cancel{x}}\cdot\frac{\cancel{x}}{x+y}=$$

$$=\frac{x+y}{x+y}=1$$

4. $$\frac{x^{3}+y^{3}}{(x+y)(x^{2}-y^{2})}+\frac{2y}{x+y}-\frac{xy}{x^{2}-y^{2}}=$$

$$=\frac{(x+y)(x^{2}-xy+y^{2})}{(x+y)^{2}(x-y)}+\frac{2y}{x+y}-\frac{xy}{(x-y)(x+y)}=$$

$$=\frac{(x^{2}-xy+y^{2})(x-y)+2y(x-y)(x+y)-xy(x+y)}{(x+y)^{2}(x-y)}=$$

$$=\frac{x^{3}+y^{3}+2x^{2}y-2y^{3}-x^{2}y-xy^{2}}{(x+y)^{2}(x-y)}=\frac{x^{3}-y^{3}+x^{2}y-xy^{2}}{(x+y)^{2}(x-y)}=$$

$$= \frac{(x-y)(x^2+xy+y^2)+xy(x-y)}{(x+y)^2(x-y)} =$$

$$= \frac{(x-y)(x^2+2xy+y^2)}{(x+y)^2(x-y)} = \frac{(x-y)(x+y)^2}{(x+y)^2(x-y)} = 1$$

5. $$\left(\frac{x}{x^2-4} - \frac{8}{x^2+2x}\right)\left(\frac{x-4}{2x-x^2}\right)^{-1} + \frac{x+8}{x+2} =$$

$$= \left(\frac{x}{(x-2)(x+2)} - \frac{8}{x(x+2)}\right)\cdot\left(\frac{x(2-x)}{x-4}\right) + \frac{x+8}{x+2} =$$

$$= \frac{x^2-8(x-2)}{x(x-2)(x+2)} \cdot \frac{x(2-x)}{x-4} + \frac{x+8}{x+2} =$$

$$= \frac{x^2-8x+16}{\cancel{x(x-2)}(x+2)} \cdot \frac{\cancel{-x(x-2)}}{x-4} + \frac{x+8}{x+2} =$$

$$= \frac{(x-4)^{\cancel{2}}}{x+2}\cdot\frac{-1}{\cancel{x-4}} + \frac{x+8}{x+2} =$$

$$= \frac{4-x}{x+2} + \frac{x+8}{x+2} = \frac{4-\cancel{x}+\cancel{x}+8}{x+2} = \frac{12}{x+2}$$

6. $$\left(\frac{x\sqrt{x}-y\sqrt{y}}{x-\sqrt{x}\sqrt{y}} - \frac{x-y}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}\sqrt{y}+2x}{3x}\right)^{-1} =$$

$$= \left(\frac{x\sqrt{x}-y\sqrt{y}}{\sqrt{x}(\sqrt{x}-\sqrt{y})} - \frac{(\sqrt{x}-\sqrt{y})\cancel{(\sqrt{x}+\sqrt{y})}}{\cancel{\sqrt{x}+\sqrt{y}}}\right)\cdot\frac{3x}{\sqrt{xy}+2x} =$$

$$= \frac{x\sqrt{x}-y\sqrt{y}-\sqrt{x}(\sqrt{x}-\sqrt{y})^2}{\cancel{\sqrt{x}}(\sqrt{x}-\sqrt{y})}\cdot\frac{3(\cancel{\sqrt{x}})^{\cancel{2}}}{\cancel{\sqrt{x}}(\sqrt{y}+2\sqrt{x})} =$$

$$= \frac{x\sqrt{x}-y\sqrt{y}-\sqrt{x}(x-2\sqrt{xy}+y)}{\sqrt{x}-\sqrt{y}}\cdot\frac{3}{\sqrt{y}+2\sqrt{x}} =$$

$$= \frac{\cancel{x\sqrt{x}} - y\sqrt{y} - \cancel{x\sqrt{x}} + 2x\sqrt{y} - y\sqrt{x}}{\sqrt{x}-\sqrt{y}} \cdot \frac{3}{\sqrt{y}+2\sqrt{x}} =$$

$$= \frac{\sqrt{y}\,(-y+2x-\sqrt{xy})}{\sqrt{x}-\sqrt{y}} \cdot \frac{3}{\sqrt{y}+2\sqrt{x}} =$$

$$= \frac{3\sqrt{y}\,(-y+2x-\sqrt{xy})}{\sqrt{xy}+2x-y-2\sqrt{xy}} = \frac{3\sqrt{y}\,(-y+2x-\sqrt{xy})}{-y+2x-\sqrt{xy}} = 3\sqrt{y}$$

7. $$\left(\sqrt{\frac{1}{x^2}-1} - \frac{1}{x}\right) \cdot \left(\frac{1-x}{\sqrt{1-x^2}-1+x} + \frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right) =$$

$$= \left(\sqrt{\frac{1-x^2}{x^2}} - \frac{1}{x}\right) \left(\frac{(\sqrt{1-x})^2}{\sqrt{1-x}\sqrt{1+x} - (\sqrt{1-x})^2} + \frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right) =$$

$$= \frac{\sqrt{1-x}\,\sqrt{1+x}-1}{x} \cdot \left(\frac{(\sqrt{1-x})^{\cancel{2}}}{\cancel{\sqrt{1-x}}\,(\sqrt{1+x}-\sqrt{1-x})} + \frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right) =$$

$$= \frac{\sqrt{1-x}\,\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1-x}+\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}} =$$

$$= \frac{(1-x)\sqrt{1+x} + (1+x)\sqrt{1-x} - \sqrt{1-x} - \sqrt{1+x}}{x(\sqrt{1+x}-\sqrt{1-x})} =$$

$$= \frac{\cancel{\sqrt{1+x}} - x\sqrt{1+x} + \cancel{\sqrt{1-x}} + x\sqrt{1-x} - \cancel{\sqrt{1-x}} - \cancel{\sqrt{1+x}}}{x(\sqrt{1+x}-\sqrt{1-x})} =$$

$$= \frac{x\,\cancel{(\sqrt{1-x}-\sqrt{1+x})}}{-x\,\cancel{(\sqrt{1-x}-\sqrt{1+x})}} = -1$$

8. $$\frac{\frac{1}{\sqrt{a-1}} + \sqrt{a+1}}{\frac{1}{\sqrt{a+1}} - \frac{1}{\sqrt{a-1}}} : \frac{\sqrt{a+1}}{(a-1)\sqrt{a+1} - (a+1)\sqrt{a-1}} =$$

$$= \frac{\frac{1+\sqrt{a^2-1}}{\sqrt{a-1}}}{\frac{\sqrt{a-1}+\sqrt{a+1}}{\sqrt{a^2-1}}} \cdot \frac{(a-1)\sqrt{a+1} - (a+1)\sqrt{a-1}}{\sqrt{a+1}} =$$

$$= \frac{\sqrt{a^2-1}\,(1+\sqrt{a^2-1})}{\sqrt{a-1}\,(\sqrt{a-1}+\sqrt{a+1})} \cdot \frac{\sqrt{a-1}\cdot\sqrt{a-1}\cdot\sqrt{a-1}+\sqrt{a+1}\,\sqrt{a+1}\,\sqrt{a-1}}{\sqrt{a+1}} =$$

$$= \frac{\cancel{\sqrt{a-1}}\,\cancel{\sqrt{a+1}}\,(1+\sqrt{a-1}\,\sqrt{a+1})}{\cancel{\sqrt{a-1}}\,(\sqrt{a-1}+\sqrt{a+1})} \cdot \frac{\sqrt{a-1}\,(\sqrt{a-1}\,\sqrt{a-1}+\sqrt{a+1}\,\sqrt{a+1})}{\cancel{\sqrt{a+1}}} =$$

$$= \frac{\sqrt{a-1}\,(1+\sqrt{a^2-1})\cdot\sqrt{a+1}\,\cancel{(\sqrt{a-1}+\sqrt{a+1})}}{\cancel{(\sqrt{a-1}+\sqrt{a+1})}} =$$

$$= \sqrt{a^2-1}\,(1+\sqrt{a^2-1}) = \sqrt{a^2-1}+a^2-1$$

9. $$\left(\frac{1}{m-\sqrt{m\cdot n}}+\frac{1}{m+\sqrt{m\cdot n}}\right)\cdot\frac{m^3-n^3}{m^2+m\cdot n+n^2} =$$

$$= \frac{m+\cancel{\sqrt{m\cdot n}}+m-\cancel{\sqrt{m\cdot n}}}{m^2-m\cdot n}\cdot\frac{(m-n)\cancel{(m^2+m\cdot n+n^2)}}{\cancel{m^2+m\cdot n+n^2}} =$$

$$= \frac{2m\,(m-n)}{m^2-m\cdot n} = \frac{2\cancel{m}\,\cancel{(m-n)}}{\cancel{m}\,\cancel{(m-n)}} = 2$$

10. $$\sqrt[3]{2+\sqrt5}+\sqrt[3]{2-\sqrt5} = x$$

$$x^3 = 2+\cancel{\sqrt5}+3\left(\sqrt[3]{2+\sqrt5}\right)^2\sqrt[3]{2-\sqrt5}+3\sqrt[3]{2+\sqrt5}\left(\sqrt[3]{2-\sqrt5}\right)^2+2-\cancel{\sqrt5} =$$

$$= 4+3\sqrt[3]{2+\sqrt5}\,\sqrt[3]{2-\sqrt5}\,\underbrace{\left(\sqrt[3]{2+\sqrt5}+\sqrt[3]{2-\sqrt5}\right)}_{x}$$

$$x^3 = 4+3\sqrt[3]{4-\cancel{2\sqrt5}+\cancel{2\sqrt5}-5}\cdot x$$

$$x^3 = 4+3\sqrt[3]{(-1)}\,x$$

$$x^3 = 4-3x$$

$$x^3+3x-4 = 0 \quad /:(x-1)$$ (Bezuov stav, nula polinoma je u $x=1$, što znači da je deljiv sa $(x-1)$.)

$$(x^3+3x-4):(x-1) = x^2+x+4$$

$$\begin{array}{l} -\cancel{x^3}+x^2 \\ \hline \cancel{x^2}+3x \\ -\cancel{x^2}+x \\ \hline 4x-4 \\ -4x+4 \\ \hline / \end{array}$$

$(x^2+x+4)(x-1)=0$

$x=1 \quad \vee \quad x^2+x+4=0$

$x_{1,2}=\frac{-1\pm\sqrt{1-16}}{2} \notin R$ $D<0$, jednadžba nema realnih rešenja

R: $\underline{\underline{x=1}}$

11. $a=\sqrt{3}-\sqrt[3]{2}$

$$\frac{a^2-2\sqrt{3}a+3-\sqrt[3]{4}}{a-\sqrt{3}}=\frac{3-2\sqrt{3}\cdot\sqrt[3]{2}+(\sqrt[3]{2})^2-2\sqrt{3}(\sqrt{3}-\sqrt[3]{2})+3-\sqrt[3]{4}}{\cancel{\sqrt{3}}-\sqrt[3]{2}-\cancel{\sqrt{3}}}=$$

$$=\frac{\cancel{6}-\cancel{2\sqrt{3}\sqrt[3]{2}}+\cancel{\sqrt[3]{4}}-\cancel{6}+\cancel{2\sqrt{3}\sqrt[3]{2}}-\cancel{\sqrt[3]{4}}}{\sqrt[3]{2}}=$$

$$=\frac{0}{\sqrt[3]{2}}=0$$

12. $x=\frac{1}{2+\sqrt{3}} \qquad y=\frac{1}{2-\sqrt{3}}$

$1+x=1+\frac{1}{2+\sqrt{3}}=\frac{2+\sqrt{3}+1}{2+\sqrt{3}}=\frac{3+\sqrt{3}}{2+\sqrt{3}}$

$1+y=1+\frac{1}{2-\sqrt{3}}=\frac{2-\sqrt{3}+1}{2-\sqrt{3}}=\frac{3-\sqrt{3}}{2-\sqrt{3}}$

$$(1+x)^{-1}+(1+y)^{-1}=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{3-\sqrt{3}}=\frac{(2+\sqrt{3})(3-\sqrt{3})+(2-\sqrt{3})(3+\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}=$$

$$=\frac{6-\cancel{2\sqrt{3}}+\cancel{3\sqrt{3}}-\cancel{3}+6+\cancel{2\sqrt{3}}-\cancel{3\sqrt{3}}-\cancel{3}}{9-3}=\frac{6}{6}=1$$

13. $a=\sqrt{\frac{x}{4y}}-\sqrt{\frac{y}{4x}}$

$$a^2=\frac{x}{4y}-2\sqrt{\frac{\cancel{x}}{4\cancel{y}}\cdot\frac{\cancel{y}}{4\cancel{x}}}+\frac{y}{4x}=\frac{x}{4y}-2\sqrt{\frac{1}{16}}+\frac{y}{4x}=\frac{x}{4y}-2\cdot\frac{1}{4}+\frac{y}{4x}=$$

$$=\frac{x}{4y}-\frac{1}{2}+\frac{y}{4x}=\frac{4x^2-8xy+4y^2}{16xy}=\frac{\cancel{4}(x^2-2xy+y^2)}{\underset{4}{\cancel{16}}xy}=\frac{(x-y)^2}{4xy}$$

$$a^2+1=\frac{x^2-2xy+y^2}{4xy}+1=\frac{x^2-2xy+y^2+4xy}{4xy}=\frac{x^2+2xy+y^2}{4xy}=\frac{(x+y)^2}{4xy}$$

$$\frac{2x\sqrt{a^2+1}}{a+\sqrt{a^2+1}}=\frac{\cancel{2}x\cdot\frac{x+y}{\cancel{2}\sqrt{xy}}}{\sqrt{\frac{x}{4y}}-\sqrt{\frac{y}{4x}}+\frac{x+y}{2\sqrt{xy}}}=$$

$$=\frac{\sqrt{x}\cancel{\sqrt{x}}\cdot\frac{x+y}{\cancel{\sqrt{x}}\sqrt{y}}}{\frac{\sqrt{x}}{2\sqrt{y}}-\frac{\sqrt{y}}{2\sqrt{x}}+\frac{x+y}{2\sqrt{xy}}}=$$

$$\frac{\frac{\sqrt{x}(x+y)}{\cancel{\sqrt{y}}}}{\frac{x-\cancel{y}+x+\cancel{y}}{2\sqrt{x}\cancel{\sqrt{y}}}}=\frac{\cancel{2x}(x+y)}{\cancel{2x}}=x+y$$

14. $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ $\quad y=\frac{2-\sqrt{3}}{2+\sqrt{3}}$

$$x+\frac{1}{x}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{(\sqrt{3}-\sqrt{2})^2+(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}=$$

$$\frac{3-\cancel{2\sqrt{6}}+2+3+\cancel{2\sqrt{6}}+2}{3-2}=10$$

$$y+\frac{1}{y}=\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{2+\sqrt{3}}{2-\sqrt{3}}=\frac{(2-\sqrt{3})^2+(2+\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})}=$$

$$\frac{4-\cancel{4\sqrt{3}}+3+4+\cancel{4\sqrt{3}}+3}{4-3}=14$$

$$\sqrt{x+\frac{1}{x}+y+\frac{1}{y}}=\sqrt{10+14}=\sqrt{24}=2\sqrt{6}$$

15. $a=2b$

$$\frac{a^4}{a^2b^2-b^4}\left(\left(\frac{b}{b-a}\right)^{-2}-\frac{(a+b)^2-4ab}{a^2-ab}\right)=$$

$$\frac{16b^4}{4b^4-b^4}\left(\left(\frac{b-2b}{b}\right)^2-\frac{(2b+b)^2-4\cdot 2b\cdot b}{4b^2-2b\cdot b}\right)=$$

$$\frac{16\cancel{b^4}}{3\cancel{b^4}}\left((-1)^2-\frac{9b^2-8b^2}{2b^2}\right)$$

$$=\frac{16}{3}\cdot\left(1-\frac{1}{2}\right)=\frac{\cancel{16}^{\,8}}{3}\cdot\frac{1}{\cancel{2}}=\frac{8}{3}$$

16. $\sqrt{(1-\sqrt{3})^2}-\sqrt{(1+\sqrt{3})^2}=$

$$=|1-\sqrt{3}|-|1+\sqrt{3}|=\cancel{\sqrt{3}}-1-1-\cancel{\sqrt{3}}=-2$$

17. $\sqrt{(\sqrt{7}-3)^2}+\sqrt[3]{(\sqrt{7}-3)^3}=$

$$=|\sqrt{7}-3|+\sqrt{7}-3=3-\sqrt{7}+\sqrt{7}-3=0$$

18.

$$(1-a^2):\left(\left(\frac{1-a^{\frac{3}{2}}}{1-a^{\frac{1}{2}}}+a^{\frac{1}{2}}\right)\cdot\left(\frac{1+a^{\frac{3}{2}}}{1+a^{\frac{1}{2}}}-a^{\frac{1}{2}}\right)\right)+1=$$

$$= (1-a^2) : \left(\frac{\left(a^{\frac{1}{2}}(1-a)+(1-a)\right)\cdot\left(a^{\frac{1}{2}}(a-1)-(a-1)\right)}{1-a} \right) + 1 =$$

$$= (1-a^2) : \left(\frac{(\sqrt{a}+1)(1-a)(\sqrt{a}-1)(a-1)}{1-a} \right) + 1 =$$

$$= \cancel{(1-a)}(1+a) \cdot \frac{(1-a)}{(a-1)^2\,\cancel{(1-a)}} + 1$$

$$= \frac{1-\cancel{a^2}+\cancel{a^2}-2a+1}{(a-1)^2} = \frac{-2a+2}{(a-1)^2} = \frac{-2(a-1)}{(a-1)^2} = \frac{-2}{a-1} = \frac{2}{1-a}$$

19. $$\frac{x^4}{x^2y^2-y^4} \cdot \left(\left(\frac{y}{y-x}\right)^{-2} - \frac{(x+y)^2-4xy}{x^2-xy} \right) =$$

$$= \frac{x^4}{y^2(x^2-y^2)} \left(\frac{(y-x)^2}{y^2} - \frac{x^2+2xy+y^2-4xy}{x(x-y)} \right)$$

$$= \frac{x^4}{y^2(x-y)(x+y)} \left(\frac{(y-x)^2 \cdot x(x-y) - (x-y)^2 y^2}{xy^2(x-y)} \right) =$$ $* \, (x-y)^2 = (y-x)^2$!

$$= \frac{\cancel{x^4}\,^{3}}{y^2(x-y)(x+y)} \cdot \frac{(x-y)^2\left(x(x-y)-y^2\right)}{\cancel{x}y^2(x-y)} =$$

$$= \frac{x^3\,\cancel{(x-y)^2}\,(x^2-xy-y^2)}{y^4\,\cancel{(x-y)^2}\,(x+y)} = \frac{x^3\,(x^2-xy-y^2)}{y^4\,(x+y)}$$